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Chiral Perturbation Theory in a Nuclear Background

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Abstract

We propose a novel way to formulate Chiral Perturbation Theory in a nuclear background, characterized by a static, non-uniform distribution of the baryon number that describes the *finite* nucleus. In the limiting case of a uniform distribution, the theory reduces to the well-known zero-temperature in-medium ChPT. The proposed approach is used to calculate the self-energy of the charged pion in the background of the heavy nucleus at $O(p^5)$ in the chiral expansion, and to derive the leading terms of the pion-nucleus optical potential.

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1 Introduction

The topic of low-energy pion-nucleus interactions has a long history and a well-developed phenomenology [1, 2, 3]. Energy spectra of pionic atoms have traditionally played an important role in promoting the understanding of this field by providing a large and systematic data base which sets tight constraints on the parameterization of the pion-nuclear potential (see Ref. [4] for a state-of-the-art update).

New developments, both experimentally and theoretically, have recently revitalized this field. Accurate measurements of deeply bound ($1s$) states of pionic atoms formed with Pb and Sn isotopes [5] have triggered renewed interest in the underlying mechanisms governing S -wave pion-nucleus interactions. The quest for “fingerprints of chiral restoration” in this context has been raised [6, 7], referring to a possible in-medium reduction of the pion decay constant which is an order parameter of the spontaneously broken chiral symmetry in QCD [8]. Recent theoretical investigations [9, 10, 11, 12] based on Chiral Perturbation Theory (ChPT) have, for one part, focused on the calculation of the in-medium shift of the pion mass, to be identified with the value of the threshold S -wave π -nucleus optical potential in the center of the nucleus. Alternatively, the energy-dependent pion self-energy (polarization function) has been extrapolated, using ChPT input and the local density approximation, to calculate directly the pionic $1s$ and $2p$ level shifts and widths for Pb and Sn isotopes [13].

What all these recent investigations have in common, is their emphasis on a framework based on chiral dynamics and on the spontaneously broken chiral symmetry of low-energy QCD. However, all these analyses rely, in one way or another, on assumptions which need further systematic scrutiny and clarification:

- i) The in-medium mass of the pion is a well-defined quantity within the in-medium ChPT. The procedure of extracting the empirical value for this quantity, however, involves detailed theoretical analysis based on phenomenological potentials, introducing model dependence which is hard to control.
- ii) The wave function of a deeply bound pionic state is concentrated near the surface of the nucleus, where the variation of the proton and fermion density distributions is maximal [16]. The commonly used expansion in gradients of the local nuclear density must therefore be carefully examined.
- iii) Pion-nuclear observables should not depend on the parameterization of the interpolating pion field in the chiral Lagrangian. Yet, questions concerning off-shell ambiguity have appeared repeatedly in recent works. In Ref. [12] it is demonstrated that the off-shell ambiguity disappears in the mass (as it should of course), if one considers the systematic chiral expansion of the in-medium mass shift. It remains to prove the equivalent statement for the

bound-state mass spectrum, e.g. as obtained in the approach of Ref. [13]. Last but not least, the complete expression of the pion self-energy operator at $O(p^5)$ and $O(p^6)$, including all strong and electromagnetic isospin-breaking effects at this orders, cannot be found in the literature so far.

It is therefore evident that further progress in the description of pion-nuclear bound systems requires the formulation of the systematic ChPT framework in the presence of a *finite* nucleus. This is what our present paper is focused on. The interest in this problem is motivated by at least three reasons:

- i) It has been argued (e.g. [13]) that chiral dynamics (the specific energy dependence of low-energy pion-nucleus interactions as imposed by chiral symmetry, in combination with the approximate vanishing of the isospin-even πN amplitude) is a key ingredient in understanding the empirically observed “missing repulsion” in the S -wave pion-nucleus optical potential. This statement has to be verified on a more systematic basis.
- ii) The in-medium mass shift of the charged pion is the only quantity known to us, which is related to the two-point Green function and where the $O(p^2)$ πN electromagnetic low-energy constant (LEC) f_1 enters at lowest order independently from other unknown LECs. On the other hand, this constant is very important in the theoretical analysis [17] of the experimental results obtained by the Pionic Hydrogen Collaboration (PSI) [18] (in particular, it provides the bulk of uncertainty in the relation of the ground-state energy-level shift to the S -wave πN scattering lengths [17]). One can expect that reliable experimental information on the in-medium mass shift can, at least, test the order-of-magnitude estimates for f_1 , on which the current numerical analysis of pionic hydrogen is based [17, 19, 20].
- iii) The consistent formulation of ChPT in the background of a finite nucleus is a challenging task of its own. It opens perspectives for many applications of ChPT in nuclear physics – especially for the problems in which the finite boundaries of the system are crucial.

The problem that we are addressing can be formulated as follows. At low energy the interactions between pions, photons and nucleons are described by the conventional Lagrangian of ChPT (we concentrate here exclusively on the 2-flavor case). Let Ω be the bound state of a nucleus containing A nucleons. We aim to describe the processes $\Omega + X \rightarrow \Omega' + Y$ where X, Y represent any number of pions and photons (in this paper we consider the case where both X and Y stand for 1-pion states). In principle one can generalize the approach to cases such as pion inelastic scattering or pion absorption, where unbound nucleons are also present. This is, however, beyond the scope of the present paper.

A rigorous field theoretical treatment would require solving the nuclear bound-state (e.g. Bethe-Salpeter) equation and then calculating transition matrix elements with in- and out-going pion fields. This task is simplified considerably for heavy nuclei (in the large- A limit). In particular, in this limit the recoil of the nucleus as a whole is a $1/A$ correction and can be neglected, and one may formally set $\Omega' = \Omega$. ChPT in a nuclear background which will be referred to as “Chiral Perturbation Theory for Heavy Nuclei” (ChPT[Ⓐ], where Ⓜ denotes the heavy nucleus with the mass number A) hereafter, is then to be understood as an approximate theory in which the transitions $\Omega + X \rightarrow \Omega + Y$ are treated in terms of simpler Green functions “in the presence of the nucleus”, $G_\Omega(X \rightarrow Y)$. The process $X \rightarrow Y$ is described by chiral dynamics, and the presence of the nucleus Ω is parametrized in terms of a few phenomenologically determined functions (e.g. proton and neutron distributions) which are directly related to corresponding observables.

In-medium ChPT for a homogeneous system starts from a nuclear Fermi gas and systematically introduces interactions mediated by real or virtual pions. ChPT[Ⓐ] is a generalization for finite systems. It reduces to the standard in-medium ChPT in the limit of a uniform distribution of baryon number. Beyond that, ChPT[Ⓐ] provides a systematic framework to study pion-nuclear bound states, for which the finite volume and the surface of the nucleus are important ingredients.

ChPT[Ⓐ] imposes chiral counting rules not only on the hierarchy of pion-nucleon interactions, but also on the relevant nuclear matrix elements. The nuclear structure information required is thus limited to expectation values of only those operators whose chiral dimension is compatible with the given order in the chiral counting at which the calculations are carried out. We demonstrate explicitly how this works in the calculation of the pion self-energy at $O(p^5)$ in ChPT. To that chiral order, the complete set of leading terms in pion-nucleus optical potential (those linear in the proton and neutron densities, $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$) are generated. It is of course well known from pion-nuclear phenomenology [1, 2, 3], that rescattering and absorption terms of $O(p^6)$ and higher orders are quantitatively important. In the present paper we focus on the systematic expansion to $O(p^5)$. Further developments concerning higher orders are planned for subsequent publications.

The layout of the paper is as follows. In section 2 we consider in detail the construction of the framework: main assumptions, role of chiral symmetry and the connection to the conventional in-medium ChPT. In section 3, using the formulated framework, we obtain the complete expression for the pion self-energy operator at $O(p^5)$ in ChPT in the background of the finite nucleus. Using the latter expressions the pion-nucleus optical potential at $O(p^5)$ is derived in section 4. In section 5 we compare with the existing approaches in the literature. Conclusions are drawn in section 6.

2 The framework

2.1 S -matrix as functional of the free fields

In order to ensure that the Green functions satisfy constraints imposed by chiral symmetry at every step of the calculation, and do not depend on the parameterization of the pion field, we equip the effective Lagrangian with external c -number sources and consider the generating function in the presence of these sources. Thus, our approach is closely related to the approach of Refs. [10, 11] which uses exactly the same setting. The difference arises at the following points:

- The main difference is that our approach is not limited to the uniform baryon density distribution and takes the finite-size effects into account. In the limit of the uniform distribution, our approach will reduce to that of Refs. [10, 11].
- The electromagnetic effects are not considered in [10, 11], although the existing framework allows for accommodating them without further modification. Since in the description of the pionic atoms where our approach will be applied, the Coulomb interaction plays an important role, we include the electromagnetic effects from the beginning.
- Neglecting 4- and more-fermion interactions in Refs. [10, 11] leads to substantial technical simplifications. In our approach, we do not make this approximation in the perturbative expansion of the generating functional. Note that in ChPT there is no contribution from the 4-fermion interactions to the pion nucleus optical potential at $O(p^5)$ (see below). For the treatment of the multinucleon interactions in the kinematical region where NN pairs are close to threshold, in addition, some kind of non-perturbative resummation might be necessary due to the large NN scattering length [11].

The formulation which was considered in Refs. [9, 12, 13] does not use external sources. For this reason, in the uniform-density limit it is equivalent to our approach only on the mass shell, i.e. in the calculation of the physical observables. Moreover, since Refs. [9, 12, 13] are based on the standard HBChPT description of pion-nucleon interactions [14], the above equivalence holds up to the contributions from the non-standard counting regime (see below)¹. Finally note, that our approach which is based on constructing the S -operator in the perturbation theory and then sandwiching it by the state vectors describing particles in the medium, is formally similar to the one used in Ref. [15] – again, in the case of the uniform medium.

¹Throughout the paper we use the terminology of Ref. [11] for different momentum regimes in the Feynman integrals

We start with setting up the shorthand notation for the external sources in the generating functional

$$j \doteq \{s, p, v, a\}, \quad j_0 \doteq \{\mathcal{M}_q, 0, 0, 0\}, \quad (2.1)$$

where s, p, v, a stand for scalar, pseudoscalar, vector and axial-vector external sources, and $\mathcal{M}_q = \text{diag}(m_u, m_d)$ denotes the quark mass matrix. The Green functions are obtained from the generating functional by differentiating with respect to the pertinent external sources, and setting $j = j_0$ at the end.

The Lagrangian which we shall use to describe the system, is the standard Lagrangian of ChPT with pions, nucleons, photons and external sources in the vacuum:

$$\mathcal{L} = \mathcal{L}_\gamma + \mathcal{L}_\pi^{(p^2)} + \mathcal{L}_\pi^{(e^2)} + \mathcal{L}_\pi^{(p^4)} + \mathcal{L}_\pi^{(e^2 p^2)} + \cdots + \mathcal{L}_N^{(p)} + \mathcal{L}_N^{(p^2)} + \mathcal{L}_N^{(e^2)} + \cdots, \quad (2.2)$$

where the dots stand for the higher-order terms in the chiral expansion, for the terms of order e^4 and higher that are consistently neglected from now on, and for the terms in the Lagrangian containing four and more nucleon fields. For the convenience of the reader we collect the known expressions for the lowest-order chiral Lagrangians in appendix A.

The asymptotic (*in-* and *out-*) states of the theory include both “elementary” particles (nucleons/antinucleons, pions, photons) as well as bound systems of nucleons (stable nuclei). On the other hand, bound systems of pions with nuclei – pionic atoms – are not present in the set of the asymptotic states, because in the present study their width can not be neglected. Instead, the pionic atoms will show up as (complex) poles on the second Riemann sheet in the pertinent Green functions. In this section, however, we consider the states containing only “elementary” particles. For example, the state $|\mathbf{n}; \bar{\mathbf{n}}; \mathbf{m}; \mathbf{k}; \text{in}(\text{out})\rangle$ with n nucleons, \bar{n} antinucleons, m pions and k photons, is given by

$$|\mathbf{p}_1 s_1, \dots \mathbf{p}_n s_n; \bar{\mathbf{p}}_1 \bar{s}_1, \dots \bar{\mathbf{p}}_{\bar{n}} \bar{s}_{\bar{n}}; \mathbf{q}_1, \dots \mathbf{q}_m; \mathbf{l}_1 \varepsilon_1, \dots \mathbf{l}_k \varepsilon_k; \text{in}(\text{out})\rangle. \quad (2.3)$$

Here, s_i (\bar{s}_i) stands for the spin projection of the nucleon (antinucleon), and ε_i for the photon polarization vector. To ease notations, we suppress the indices that distinguish the different species of the nucleons (antinucleons) and pions.

The generic off-shell transition process $X \rightarrow Y$ in the presence of any number of elementary fermions and bosons in *in-* and *out-*states, with the sets X, Y both containing only bosons (pions and photons), is described in ChPT by the following matrix element

$$\langle \mathbf{n}'; \bar{\mathbf{n}}'; \mathbf{m}'; \mathbf{k}'; \text{out} | T \mathbf{o}_1(y_1) \cdots \mathbf{o}_\omega(y_\omega) \mathbf{o}_1(x_1) \cdots \mathbf{o}_\rho(x_\rho) | \mathbf{n}; \bar{\mathbf{n}}; \mathbf{m}; \mathbf{k}; \text{in} \rangle. \quad (2.4)$$

Here two sets of bilinear quark currents with pertinent quantum numbers (pseudoscalar current for pions, vector current for photons): $\mathbf{o}_1(x_1) \cdots \mathbf{o}_\rho(x_\rho)$ and $\mathbf{o}_1(y_1) \cdots \mathbf{o}_\omega(y_\omega)$ describe the bosons present in X and Y , respectively.

The expression (2.4) is given in the Heisenberg picture. We further wish to rewrite this expression in the interaction picture. This can be done in a standard manner, using the evolution operator (see, e.g. [21]). The result is given by

$$\langle n'; \bar{n}'; m'; k' | T o_1(y_1) \cdots o_\omega(y_\omega) o_1(x_1) \cdots o_\rho(x_\rho) \mathbf{S} | n; \bar{n}; m; k \rangle, \quad (2.5)$$

where \mathbf{S} denotes the conventional S -operator, $o_1(y_1) \cdots o_\omega(y_\omega) o_1(x_1) \cdots o_\rho(x_\rho)$ stand for the currents in the interaction picture, and the free-particle states $|n; \bar{n}; m; k\rangle$ are the eigenstates of the unperturbed Hamiltonian. Using Wick's theorem, the T -product in Eq. (2.5) can be expressed in terms of the normal products of free nucleon, pion and photon fields

$$\begin{aligned} \Psi(x) &= \begin{pmatrix} \Psi_p(x) \\ \Psi_n(x) \end{pmatrix}, \quad \Phi(x) = \begin{pmatrix} \Phi_+(x) \\ \Phi_0(x) \\ \Phi_-(x) \end{pmatrix}, \\ \Psi_i(x) &= \sum_s \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\sqrt{M_i^2 + \mathbf{k}}} (\mathrm{e}^{-ik \cdot x} b_i(\mathbf{k}, s) u_i(\mathbf{k}, s) + \mathrm{e}^{ik \cdot x} d_i^\dagger(\mathbf{k}, s) v_i(\mathbf{k}, s)), \\ \Phi_\alpha(x) &= \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\sqrt{m_\alpha^2 + \mathbf{k}^2}} (a_\alpha(\mathbf{k}) \mathrm{e}^{-ik \cdot x} + a_\alpha^\dagger(\mathbf{k}) \mathrm{e}^{ik \cdot x}), \\ \mathcal{A}_\mu(x) &= \sum_\lambda \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} (c_\lambda(\mathbf{k}) \varepsilon_\mu(\lambda) \mathrm{e}^{-ik \cdot x} + c_\lambda^\dagger(\mathbf{k}) \varepsilon_\mu^*(\lambda) \mathrm{e}^{ik \cdot x}). \end{aligned} \quad (2.6)$$

Here, $i = p, n$ and $\alpha = \pm, 0$ stand for different nucleon and pion species. In the subsequent expressions we shall suppress these indices, if this does not lead to the confusion. Furthermore, $u_i(\mathbf{k}, s)$, $v_i(\mathbf{k}, s)$ denote conventional free Dirac spinors and $\varepsilon_\mu(\lambda)$ is the photon polarization vector. The state vectors from Eq. (2.5) are obtained by repeated action of creation operators on the perturbative vacuum

$$|n; \bar{n}; m; k\rangle = \frac{1}{\sqrt{n! \bar{n}! m! k!}} \underbrace{b^\dagger(\mathbf{p}, s)}_n \cdots \underbrace{d^\dagger(\bar{\mathbf{p}}, \bar{s})}_{\bar{n}} \cdots \underbrace{a^\dagger(\mathbf{k})}_{m} \cdots \underbrace{c^\dagger(\mathbf{l})}_{k} \cdots |0\rangle. \quad (2.7)$$

In the canonical formalism, the free fields introduced above coincide with the interpolating field operators at time $t = 0$. At an arbitrary t , the relation between free and interpolating fields is given by the evolution operator. However, in order to avoid imposing boundary conditions on the interpolating pion field which is not uniquely defined, we reformulate the framework in terms of the functional integral depending on the external sources j . To this end, we consider the generating functional in the theory given by Lagrangian (2.2)

$$Z(j|\eta, \bar{\eta}) = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \exp \left\{ i \int (\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta) d^4x \right\}, \quad (2.8)$$

where $\mathcal{L} = \mathcal{L}(U, \psi, \bar{\psi}, A_\mu; j)$. Here U stands for the pion interpolating field matrix, A_μ is the photon field, $\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$ denotes the two-component interpolating nucleon field and $\bar{\psi} = \psi^\dagger \gamma_0$ its conjugate. $\eta = \begin{pmatrix} \eta_p \\ \eta_n \end{pmatrix}$ is the nucleon external source and $\bar{\eta}$ its conjugate. Note that the meson Green functions which are obtained by differentiating the quantity Z with respect to the bosonic external sources j , do not depend on the parameterization of the pion interpolating field U , the latter being merely an integration variable in the functional integral. Thus, the “off-shell ambiguity” mentioned in the introduction, never arises in this formulation.

At the first step, we expand the generating functional (2.8), in powers of nucleon sources $\eta, \bar{\eta}$

$$\begin{aligned} Z(j|\eta, \bar{\eta}) &= Z^{(0)}(j) + \int d^4x d^4y \bar{\eta}(x) Z^{(1)}(j|x, y) \eta(y) \\ &+ \frac{1}{2} \int d^4x_1 d^4x_2 d^4y_1 d^4y_2 \bar{\eta}(x_1) \bar{\eta}(x_2) Z^{(2)}(j|x_1, x_2, y_1, y_2) \eta(y_1) \eta(y_2) + \dots \end{aligned} \quad (2.9)$$

(Note that, since the interactions with the external sources j preserve the baryon number, we always have equal number of η 's and $\bar{\eta}$'s in this expansion.)

We further define the operator $\hat{S}(j)$ through the following construction:

$$\begin{aligned} \hat{S}(j) &= S^{(0)}(j) + \int d^4x d^4y : \bar{\Psi}(x) S^{(1)}(j|x, y) \Psi(y) : \\ &+ \frac{1}{2} \int d^4x_1 d^4x_2 d^4y_1 d^4y_2 : \bar{\Psi}(x_1) \bar{\Psi}(x_2) S^{(2)}(j|x_1, x_2, y_1, y_2) \Psi(y_1) \Psi(y_2) : + \dots \end{aligned} \quad (2.10)$$

where

$$\begin{aligned} S^{(0)}(j) &= Z^{(0)}(j), \\ S^{(1)}(j|x, y) &= z_N^{-1/2} (i \not{\partial}_x - M) Z^{(1)}(j|x, y) (-i \not{\partial}_y - M) z_N^{-1/2}, \end{aligned} \quad (2.11)$$

and so forth. Here $M = \text{diag}(M_p, M_n)$ and $z_N = \text{diag}(z_p, z_n)$ are, respectively, the 2×2 nucleon mass matrix and the nucleon wave function renormalization matrix. Furthermore, “ $: (\dots) :$ ” denotes the normal ordering of the operators. The operator $\hat{S}(j)$ has the meaning of the S -matrix defined on the subspace of the state vectors containing only fermions, in the presence of the external bosonic sources j .

In order to obtain the matrix element (2.5), we further differentiate the operator $\hat{S}(j)$ with respect to the pertinent bosonic sources, and expand the result in powers of j . This result is symbolically written as

$$\frac{(-i)^{\rho+\omega} \delta^{\rho+\omega} \hat{S}(j)}{\delta j_1(y_1) \cdots \delta j_\omega(y_\omega) \delta j_1(x_1) \cdots \delta j_\rho(x_\rho)} = \hat{S}_0^{\{Y,X\}}(y_1 \cdots y_\omega; x_1 \cdots x_\rho)$$

$$\begin{aligned}
& + \int d^4z j_\sigma(z) \hat{S}_1^{\{Y,X\},\sigma}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z) \\
& + \frac{1}{2} \int d^4z_1 d^4z_2 j_\sigma(z_1) j_\delta(z_2) \hat{S}_2^{\{Y,X\},\sigma\delta}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z_1, z_2) + \dots, \quad (2.12)
\end{aligned}$$

where the index of the external source j corresponds to the quantum numbers of the quark current: $j_\sigma \doteq s, p, v, a$. Further, in a complete analogy with Eqs. (2.9), (2.10) and (2.11), we construct the operator

$$\begin{aligned}
\bar{S}^{\{Y,X\}}(y_1 \cdots y_\omega; x_1 \cdots x_\rho) &= \hat{S}_0^{\{Y,X\}}(y_1 \cdots y_\omega; x_1 \cdots x_\rho) \\
& + \int d^4z : \Phi_\alpha(z) : \bar{S}_1^{\{Y,X\},\alpha}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z) \\
& + \int d^4z : \mathcal{A}_\mu(z) : \bar{S}_1^{\{Y,X\},\mu}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z) + \dots, \quad (2.13)
\end{aligned}$$

where, in analogy with Eq. (2.11), the coefficient $\bar{S}^{\{Y,X\}\dots}$ in (2.13) are obtained from $\hat{S}^{\{Y,X\}\dots}$ appearing in Eq. (2.12): e.g., $\bar{S}^{\{Y,X\}\alpha}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z)$ is obtained from $\hat{S}^{\{Y,X\}\alpha}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; z)$ by removing the pion pole in the Fourier transform with respect to the variable z , and multiplying the result by the wavefunction renormalization factor $\sqrt{z_{\pi^\alpha}}$. Integrating with the solution of the free-field equation $\Phi_\alpha(z)$ finally puts the 4-momentum, conjugate to the variable z , on the mass shell. Other coefficients in the expansion (2.13) are obtained similarly. We do not repeat their (evident) explicit expressions here. At the end, the operator $\bar{S}^{\{Y,X\}}(y_1 \cdots y_\omega; x_1 \cdots x_\rho)$ is expressed in terms of the normal products of all free fields $: \bar{\Psi} \cdots \Phi_\alpha \cdots \mathcal{A}_\mu \cdots \Psi :$ (we recall that $\hat{S}^{\{Y,X\}\dots}(y_1 \cdots y_\omega; x_1 \cdots x_\rho; \dots)$ are given by infinite series over normal products of fermion fields, cf. with Eq. (2.10)).

Finally, the matrix element of the T -product of the currents in the Heisenberg picture (2.4) can be rewritten in the form of a matrix element of the operator $\bar{S}^{\{Y,X\}}$ between the free-particle states

$$\langle n'; \bar{n}'; m'; k' | \bar{S}^{\{Y,X\}}(y_1 \cdots y_\omega; x_1 \cdots x_\rho) | n; \bar{n}; m, k \rangle. \quad (2.14)$$

This is the result we were looking for. The following remarks are in order:

- i) The construction presented in this section is the conventional LSZ formalism for the calculation of the matrix elements (2.4) in the language of functional integrals (see e.g. [22]). Note that here one does not impose boundary conditions on the interpolating fields at $t = 0$. This is crucial for ensuring chiral symmetry at every step, and for circumventing the problem of the “off-shell ambiguity”.
- ii) The coefficient functions of the operators $\hat{S}(j)$ and $\bar{S}^{\{Y,X\}}$ are ultraviolet-finite, since the effective Lagrangian (2.2) includes the set of counterterms

that make all S -matrix elements finite. On the other hand, $Z(j|\eta, \bar{\eta})$, in general, can not be made ultraviolet-finite with chirally-invariant counter-terms alone [25].

- iii) In the presence of photons, the coefficient functions in $\hat{S}(j)$ and $\bar{S}^{\{Y,X\}}$ are infrared-divergent. We shall regularize the infrared (as well as the ultraviolet) divergences by using the dimensional regularization. The mass-shell limit is then understood in space-time dimensions different from four.
- iv) The coefficient functions in $\hat{S}(j)$ and $\bar{S}^{\{Y,X\}}$ are, in general, gauge-dependent, since they involve off-shell legs. Only those quantities that correspond to physically observable processes should be gauge-independent.
- v) In the calculation of the coefficient functions, some prescription is implied that preserves chiral power counting in the presence of nucleons. For example, one could have chosen heavy baryon ChPT [14], or the infrared regularization method [26]. As the most general prescription, we propose to use the so-called threshold expansion of the relativistic Feynman integrals [27] which, by construction, allows to keep the contributions from all soft integration regimes. In particular, the threshold expansion will automatically produce the contributions to the Feynman integrals not only from the “HBChPT regime”, but from the “non-standard regime(s)” [11] as well. Below we assume that the threshold expansion is always applied to all Feynman diagrams, even if this is not explicitly stated.

2.2 Scattering processes in the presence of the nucleus

The asymptotic spectrum of the theory described by the Lagrangian (2.2) includes one-particle state vectors representing the nucleus as a bound system of A nucleons:

$$|\Omega; in\rangle = |\Omega; out\rangle = |\Omega\rangle, \quad \langle \Omega' | \Omega \rangle = (2\pi)^3 2P_\Omega^0 \delta^3(\mathbf{P}'_\Omega - \mathbf{P}_\Omega). \quad (2.15)$$

At this stage $|\Omega\rangle$ can still be any highly excited nuclear state as well. The scattering process $\Omega + X \rightarrow \Omega' + Y$ is described by the following matrix element:

$$\begin{aligned} & \langle \Omega' | T \mathbf{o}_1(y_1) \cdots \mathbf{o}_\omega(y_\omega) \mathbf{o}_1(x_1) \cdots \mathbf{o}_\rho(x_\rho) | \Omega \rangle \\ &= \langle \Omega' | T o_1(y_1) \cdots o_\omega(y_\omega) o_1(x_1) \cdots o_\rho(x_\rho) \mathbf{S} | \Omega \rangle. \end{aligned} \quad (2.16)$$

Here

$$|\Omega\rangle = \mathbf{U}(-\infty, 0)|\Omega\rangle, \quad \langle \Omega' | = \langle \Omega' | \mathbf{U}(0, +\infty), \quad (2.17)$$

where $\mathbf{U}(t', t)$ stands for the conventional evolution operator (see, e.g. [21]).

The basis $|n; \bar{n}; m; k\rangle$ of the eigenstates of the free Hamiltonian is complete. A general nuclear state $|\Omega\rangle$ can be written as a linear combination of the basis states

$$|\Omega\rangle = \sum_{n; \bar{n}; m; k} C_{n; \bar{n}; m; k} |n; \bar{n}; m; k\rangle. \quad (2.18)$$

From the comparison to the expression of the bound-state matrix elements in the Mandelstam formalism, one can obtain the relation to the Bethe-Salpeter amplitude for the bound state

$$C_{n; \bar{n}; m; k} = \int d^3\mathbf{x}_1 \cdots d^3\mathbf{x}_A G_{n; \bar{n}; m; k}(\mathbf{x}_1 \cdots \mathbf{x}_A) \langle 0 | \Psi(0, \mathbf{x}_1) \cdots \Psi(0, \mathbf{x}_A) | \Omega \rangle, \quad (2.19)$$

where $\Psi(t, \mathbf{x})$ denotes the nucleon field operator in the Heisenberg representation. To make the interpretation easier, we found it useful to give this relation in a form containing the equal-time amplitude, which in the non-relativistic limit reduces to the Schrödinger wave function of the nucleus. In the language that uses the external sources, the Bethe-Salpeter amplitude is defined via the residues of the pertinent Green functions, without referring to the matrix elements of the interpolating fields. Further, $G_{n; \bar{n}; m; k}(\mathbf{x}_1 \cdots \mathbf{x}_A)$ stands for the perturbative kernel, which can, in principle, be calculated in ChPT. However, in our approach one does not need the explicit knowledge of neither this kernel, nor the Bethe-Salpeter amplitude.

The quantity (2.16) which we want to calculate is given in the form of a matrix element of the operator $\bar{S}^{\{YX\}}$ introduced in the previous section, between the nuclear state vectors (2.18)

$$\langle \Omega' | T \mathbf{o}_1(y_1) \cdots \mathbf{o}_\omega(y_\omega) \mathbf{o}_1(x_1) \cdots \mathbf{o}_\rho(x_\rho) | \Omega \rangle = \langle \Omega' | \bar{S}^{\{YX\}}(y_1 \cdots y_\omega, x_1 \cdots x_\rho) | \Omega \rangle. \quad (2.20)$$

The right-hand side of this equation has the form $\sum_i S_i \langle \Omega' | : P_i : | \Omega \rangle$, where S_i denote the coefficient functions, and $: P_i :$ stand for the normal products of the free fields $:\bar{\Psi} \cdots \Phi_\alpha \cdots \mathcal{A}_\mu \cdots \Psi:$. The main idea of ChPT^② consists in the following: one calculates the coefficient functions S_i in standard ChPT, whereas the matrix elements of the normal products of the free fields are parametrized in terms of a few characteristic functions describing nuclear dynamics. This is the place where the empirical input enters in our calculational scheme². Our strategy consists in relating these matrix elements to a limited number of nuclear observables, and predicting other observables where the same matrix elements occur.

²As it stands, the approach is based on a well-defined separation of the perturbative and non-perturbative contributions in the matrix elements of currents. Using the same arguments one could, in principle, reshuffle particular contributions between these two groups, and design a different scheme. Here, we shall not explore this possibility any further.

If the approach is used to describe a large nucleus ($A \rightarrow \infty$), in its ground state, the following assumption can be justified:

Assumption 1. Matrix elements of operators containing at least one free pion or photon field, vanish between the nuclear states. We interpret this property in the following way: $|\Omega\rangle$ describes the ground state of the nucleus as a system of nucleons only at $A \rightarrow \infty$.

The above assumption enables one to rewrite the matrix element (2.16) in terms of the operator $\hat{S}(j)$ defined by Eq. (2.10), as

$$\langle \Omega' | \frac{(-i)^{\rho+\omega} \delta^{\rho+\omega} \hat{S}(j)}{\delta j_1(y_1) \cdots \delta j_\omega(y_\omega) \delta j_1(x_1) \cdots \delta j_\rho(x_\rho)} |\Omega\rangle \Big|_{j=j_0}. \quad (2.21)$$

Assumption 2. In order to simplify the present calculations, we assume that the nucleus is in the spin-saturated state. This means that the nucleus is not characterized by internal quantum numbers. The latter assumption, which can obviously be relaxed on a later stage, in combination with the static limit ($A \rightarrow \infty$), helps to reduce significantly – on the basis of symmetry considerations alone – the amount of empirical input needed on the nuclear side.

Assumption 3. Finally, we assume that the nuclear background does not affect the ultraviolet properties of the theory. In the case of the infinite nuclear medium, this is an exact statement, since all diagrams with medium insertions contain a cutoff at Fermi-momentum k_F . The generalization to finite systems amounts to require that Fourier transforms of distributions involving $|\Omega\rangle$ drop sufficiently rapidly at large momentum. Put differently, we assume that in the expansion $\sum_i S_i \langle \Omega' | : P_i : |\Omega\rangle$ each matrix element is finite, and there are no cancellations of divergences between various terms (we recall that S_i are finite after renormalization).

It should be emphasized that, with the above assumptions, ChPT[®] can be pursued to any given chiral order. The consistency of the approach, and the validity of the above assumptions, should then be verified *a posteriori* order by order in the chiral expansion.

2.3 Pion-nucleus bound state

The primary aim of the present paper is to study states of negatively charged pions bound to heavy nuclei. To this end, it is useful to consider the two-point function of the pseudoscalar densities, sandwiched by the nuclear states

$$\bar{D}_{\Omega'\Omega}(q', q) = \frac{-i}{(2BF)^2} \int d^4x d^4y e^{iq'\cdot x - iq\cdot y} \langle \Omega' | \frac{\delta^2 \hat{S}(j)}{\delta p^+(x) \delta p^-(y)} |\Omega\rangle \Big|_{j=j_0}, \quad (2.22)$$

where F is the pion decay constant in the chiral limit, B is related to the quark condensate ($F^2 B = -\lim_{m_u, m_d \rightarrow 0} \langle \bar{q}q \rangle$). We follow the Condon-Shortley phase convention and express the isovector (pseudoscalar) source field as $p^\pm(x) =$

$(2)^{-1/2}(\mp p_1(x) + ip_2(x))$, $p^0(x) = p_3(x)$. The normalization in Eq. (2.22) is defined so that in lowest order of the chiral expansion, the residue in the pole of this two-point function is normalized to unity.

Using translational invariance, we remove an overall δ -function, corresponding to the conservation of the total 3-momentum, from the quantity $\bar{D}_{\Omega'\Omega}(q', q)$. In the remaining matrix element one may pass to the static limit $P'_\Omega = P_\Omega = M_\Omega(1, \mathbf{0})$. In the calculations, it is convenient to use the Hilbert space where the static nucleus is described by the state vector $|\Omega\rangle$ normalized to unity³. Using the new notations, the right-hand side of Eq. (2.22) at $A \rightarrow \infty$ can be written as

$$\begin{aligned}\bar{D}_{\Omega'\Omega}(q', q) &= (2\pi)^3 \delta^3(\mathbf{P}'_\Omega + \mathbf{q}' - \mathbf{P}_\Omega - \mathbf{q}) \bar{D}(q', q), \\ \bar{D}(q', q) &= \frac{-i}{(2BF)^2} \int d^4x d^4y e^{iq' \cdot x - iq \cdot y} (\Omega | \frac{\delta^2 \hat{S}(j)}{\delta p^+(x)\delta p^-(y)} | \Omega) \Big|_{j=j_0},\end{aligned}\quad (2.23)$$

For a static nuclear background, energy conservation implies

$$\bar{D}(q', q) = 2\pi \delta(q^{0'} - q^0) D(E; \mathbf{q}', \mathbf{q}), \quad E = q^{0'} = q^0. \quad (2.24)$$

The pion self-energy is defined from the inverse of the operator D as follows:

$$\Pi(E; \mathbf{q}', \mathbf{q}) = (2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q}) D_0^{-1}(E; \mathbf{q}^2) - D^{-1}(E; \mathbf{q}', \mathbf{q}), \quad (2.25)$$

where $D_0^{-1}(E; \mathbf{q}^2) = E^2 - \mathbf{q}^2 - m_\pi^2$ is the inverse of the free pion propagator, and $m_\pi \doteq m_{\pi^+}$ stands for the physical mass of the charged pion in the vacuum.

Our strategy for finding the energy levels and widths of the pionic bound state implies the following steps:

1. Calculate the non-local pion self-energy operator $\Pi(E; \mathbf{q}', \mathbf{q})$ in the systematic ChPT expansion.
2. Find the pole position in the two-point function $D(E; \mathbf{q}', \mathbf{q})$ with a given self-energy operator $\Pi(E; \mathbf{q}', \mathbf{q})$, using non-perturbative methods. The real and imaginary parts of the pole position are related to the real energy and decay width of this state.
3. At the last step, for a given self-energy operator calculated perturbatively at a fixed order in ChPT, study the chiral expansion of the energy and the decay width. This enables one to control the convergence in the bound-state characteristics at all stages of calculations, possibly eliminating (potentially large) higher-order contributions that arise due to the use of the non-perturbative techniques.

³This state vector, which describes the nucleus in the static limit, is in fact defined by Eq. (2.23). The more detailed treatment of the static limit is considered below, in section 2.6.

It should be understood that identifying the (complex) bound-state energy with the pole position of the pion two-point function in the nuclear background (2.24) implies additional assumptions:

Assumption 4. In Eq. (2.22) the nucleus is on shell in its ground state, whereas in the pion-nucleus bound state both pion and nucleus are off shell. The rigorous definition of the bound-state energy involves finding the pole position of the Green function of $2A$ fermion and two pion fields. We expect, however, that for a very large A one may safely put the nucleus on mass shell.

Assumption 5. If the nucleus is put on shell, this means that the effect coming from nuclear excited states is neglected. In order to demonstrate this, note that the initial-/final- state interactions in the A ingoing/outgoing nucleon legs of any Green function create a tower of radial-excited nuclear bound states. Putting the nuclear momentum on the mass shell eliminates all but the ground state contribution to the Green function and thus, from the mathematical viewpoint, amounts to "freezing" the nucleus in its ground state during the whole interaction with the projectile [23, 24].

By construction, the energy of the bound state obtained with this procedure, does not have any spurious dependence on the choice of parameterization for the pion field U . If, instead of the quantity \bar{D} given by Eq. (2.22), one considers the two-point function of the interpolating pion field, such a dependence arises in this two-point function itself [12, 13]. As one would expect, this dependence indeed disappears in the pole position up to a given order in the chiral expansion [12]. When solving the bound-state equation numerically for the non-local self-energy [13], it is in general difficult to demonstrate that the dependence on the parameterization of the pion field is of higher chiral order. It is therefore advantageous to work in a framework which avoids this dependence *ab initio* [of course, the pole positions in the two-point functions of the pseudoscalar densities, and of the pion fields on the other hand, should coincide order by order in the chiral expansion].

We finally note that the two-point function is gauge-dependent. In order to demonstrate that the approach is consistent at a given chiral order, one has to explicitly show that the pole position is invariant up to this order. In this paper, we check the gauge invariance up to and including $O(p^5)$ (see section 3). At present, we are however not aware of the general proof of the gauge invariance that extends to all orders within our approach. If it turns out at higher orders, that the gauge invariance of the bound-state energy is lost, this would signal about the inadequacy of the approximations which were used in the formulation of the present framework. From the phenomenological point of view, the issue is totally irrelevant, since it concerns only the higher-order electromagnetic corrections to the pion-nuclear optical potential.

2.4 Free Fermi-gas

In order to examine the limit of a homogeneous nuclear medium, it is instructive to start with the simple case of a Fermi gas. The ground state of this system has all levels filled with nucleons up to the Fermi-momentum k_F . To simplify notations, we assume here in addition that isospin symmetry is not broken, switch off the electromagnetic interactions, and suppress the index labeling different nucleon species p and n . The ground state in this approximation is

$$|\Omega\rangle \rightarrow |\Omega_0\rangle = \mathcal{N} \prod_{\mathbf{l}_i, s_i=\pm 1/2}^{k_F} b^\dagger(\mathbf{l}_i, s_i) |0\rangle, \quad (2.26)$$

where $|0\rangle$ denotes the perturbative vacuum, and \mathcal{N} is a normalization constant. Furthermore, we again introduce the “round-bracket” notation, with $(\Omega_0|\Omega_0) = 1$ [see section 2.6 for further details.].

Using the canonical commutation relations, one can easily show that

$$\begin{aligned} (\Omega_0|b^\dagger(\mathbf{l}_1, s_1)b(\mathbf{l}_2, s_2)|\Omega_0) &= (2\pi)^3 2l_1^0 \delta^3(\mathbf{l}_1 - \mathbf{l}_2) \delta_{s_1 s_2} \theta(k_F - |\mathbf{l}_1|), \\ (\Omega_0|b^\dagger(\mathbf{l}_1, s_1)b^\dagger(\mathbf{l}_2, s_2)b(\mathbf{l}_3, s_3)b(\mathbf{l}_4, s_4)|\Omega_0) &= (2\pi)^3 2l_1^0 \delta^3(\mathbf{l}_1 - \mathbf{l}_4) \delta_{s_1 s_4} \theta(k_F - |\mathbf{l}_1|) (2\pi)^3 2l_2^0 \delta^3(\mathbf{l}_2 - \mathbf{l}_3) \delta_{s_2 s_3} \theta(k_F - |\mathbf{l}_2|) \\ &\quad - [perm. 1 \leftrightarrow 2], \end{aligned} \quad (2.27)$$

and so forth. The free fermion (nucleon) propagator in momentum space, with Ω_0 in the background, is given by

$$\begin{aligned} S_{\Omega_0}(p) &= i \int d^4x e^{ip \cdot x} (\Omega_0|T\Psi(x)\bar{\Psi}(0)|\Omega_0) \\ &= i \int d^4x e^{ip \cdot x} \langle 0|T\Psi(x)\bar{\Psi}(0)|0\rangle + i \int d^4x e^{ip \cdot x} (\Omega_0|:\Psi(x)\bar{\Psi}(0):|\Omega_0) \\ &= (M_\Psi + p) \left(\frac{1}{M_\Psi^2 - p^2 - i0} - 2\pi i \delta(p^2 - M_\Psi^2) \theta(p^0) \theta(k_F - |\mathbf{p}|) \right), \end{aligned} \quad (2.28)$$

where M_Ψ stands for the mass of the fermion field. One immediately recognizes that the quantity $S_{\Omega_0}(p)$ is the relativistic in-medium free fermion propagator.

In the calculation of the Green functions of the external sources j , one needs to evaluate only the closed fermion lines at non-zero fermion density (see Fig. 1). Let us first consider the closed loop with two vertices $\Gamma_{1,2}$, where these stand for arbitrary matrices in the space of Dirac and flavor indices, describing the coupling of the external sources j to the fermion bilinears. Applying Wick’s theorem and using Eq. (2.27) for the normal products, we get

$$\begin{aligned} &i^2 (\Omega_0|T\bar{\Psi}(x)\Gamma_1\Psi(x)\bar{\Psi}(y)\Gamma_2\Psi(y)|\Omega_0) \\ &= -\text{Tr}[S_{\Omega_0}(x-y)\Gamma_2 S_{\Omega_0}(y-x)\Gamma_1] + \text{Tr}[S_{\Omega_0}(0)\Gamma_2] \text{Tr}[S_{\Omega_0}(0)\Gamma_1]. \end{aligned} \quad (2.29)$$

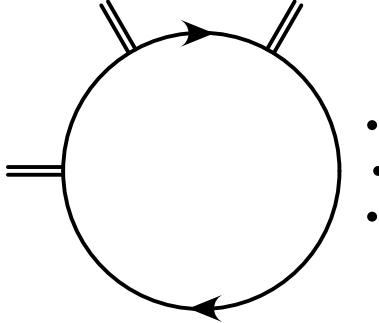


Figure 1: Closed fermion loop with any number of vertices $\Gamma_1, \Gamma_2, \dots$. Double lines denote the external sources j attached to the loop.

The above formula states that the closed fermion loop with two vertices in the background of Ω_0 is calculated replacing free nucleon propagators by the in-medium fermion propagators $S_{\Omega_0}(x)$. The same rule holds for a loop with any number of vertices, or any number of factorized loops. Indeed, we may redefine the normal product with respect to the new vacuum $|\Omega_0\rangle$. Then, according to Eq. (2.28), the fermion propagator is just $S_{\Omega_0}(x)$. Since the combinatorial factors, produced by the contractions in the Wick's theorem, do not depend on the explicit form of the propagator, one finds that all differences between closed fermion loops calculated in the vacuum and in the Ω_0 background reduce to the replacement of the nucleon propagator in the vacuum by $S_{\Omega_0}(x)$. One concludes that, for a uniform fermion density distribution Ω_0 , the approach described in the present paper reduces to the standard in-medium ChPT.

2.5 Chiral power counting

Another crucial ingredient for the approach proposed here is the systematic book-keeping based on chiral symmetry. On first sight, the formalism allows too much freedom in the nuclear matrix elements, which have to be evaluated for arbitrary space-time arguments of the fields $\Psi(x)$. We shall now demonstrate that the chiral power counting, in general, enables one to substantially reduce this freedom.

Even for a non-uniform distribution of baryon number, it is still useful to think in terms of the “local Fermi momentum k_F ” which in this case is a function of the space coordinate. It is common and convenient to count k_F at $O(p)$ [the actual numerical value for this quantity in heavy nuclei, $k_F \sim 2m_\pi$, supports this conjecture]. Consequently, the kinetic energies of the nucleons can be counted as $O(p^2)$. The fermion field itself is $O(p^{3/2})$, $|\Omega| \sim O(1)$, and the counting in the coefficient functions $S^{(n)}$ is standard. Furthermore, in order to be conform with the chiral power counting for the pion self-energy operator (2.25) in the uniform medium, we declare this quantity to be calculated at $O(p^k)$, if its actual

power according to the above counting is $k - 3$ (we remind the reader that, for the uniform distribution, the self-energy operator reduces to $\Pi(E; \mathbf{q}', \mathbf{q}) \rightarrow (2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q}) \tilde{\Pi}(E; \mathbf{q}^2)$, and k thus corresponds to the chiral power of $\tilde{\Pi}(E; \mathbf{q}^2)$, whereas three additional powers are “eaten” by the δ -function.).

The chiral power counting described here implies a systematic expansion of the matrix elements in Taylor series, so that finally one ends up with the matrix elements with fields $\Psi(x)$ and derivatives thereof, taken at coinciding space-time arguments. Such matrix elements can in turn be written in terms of local distributions. This makes their construction transparent and reliable.

Below we shall schematically explain this method for the case of the normal product of two fermion fields. The corresponding contribution to the π -nucleus scattering matrix element is given by

$$\bar{D}^{(1)}(q', q) = \int d^4x d^4y D_{ba}^{(1)}(q', q|x, y) (\Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega) , \quad (2.30)$$

where

$$D_{ba}^{(1)}(q', q|x, y) = \frac{-i}{(2BF)^2} \int d^4u d^4z e^{iq' \cdot u - iq \cdot z} \frac{\delta^2 S_{ba}^{(1)}(j|x, y)}{\delta p^+(u) \delta p^-(z)} \Big|_{j=j_0} , \quad (2.31)$$

and $a = (\alpha, i)$, $b = (\beta, j)$ stand for multiindices with α, β and i, j denoting Lorentz and isospin indices, respectively. Furthermore, the threshold expansion is implied only for the quantity $S^{(1)}$ which is calculated in the vacuum. There is no need to perform the threshold expansion in the matrix element $(\Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega)$, since, according to our picture of the nucleus, all 3-momenta involved receive a cutoff at k_F .

Let us first consider the case of the standard HBChPT counting. In this case, both energy and 3-momentum of the pions in the Feynman diagram count like $O(p)$. In order to express the nuclear matrix elements in terms of local distributions, we introduce the center-of-mass and the relative coordinates $x = R + s/2$ and $y = R - s/2$. Since the nucleus is static, the matrix element does not depend on the time component R^0 of R . Now, we consider the dependence of this quantity on s^0 . Factoring out the nucleon (proton or neutron) mass M_Ψ , which is counted at $O(1)$, we can expand the rest in the Taylor series of s^0

$$\begin{aligned} (\Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega) &= e^{iM_\Psi s^0} (\Omega | : \bar{N}_b(0, \mathbf{x}) N_a(0, \mathbf{y}) : | \Omega) \\ &- e^{iM_\Psi s^0} \frac{s^0}{2} (\Omega | : \bar{N}_b(0, \mathbf{x}) \overset{\leftrightarrow}{\partial}_0 N_a(0, \mathbf{y}) : | \Omega) + \dots , \end{aligned} \quad (2.32)$$

where $N_a(x) \doteq e^{iM_\Psi x^0} \Psi_a(x)$. Every time derivative acting on $N_a(x)$ produces the kinetic energy of the nucleon, and has therefore to be counted as a quantity of order p^2 . On the other hand, every power of s^0 corresponds to the differentiation with respect to the energy variable in the Fourier transform of $S^{(1)}$. Since,

according to the standard HBChPT counting, differentiation of every fermion propagator reduces the chiral power by 1, one may count s^0 at $O(p^{-1})$. We finally see that every term in the expansion (2.32) is $O(p)$ as compared to the previous term, so, at a given order in the chiral expansion, one may truncate the series, and arrives at a result that is local in the relative time variable s^0 .

It is clear that this argumentation does not work for the spatial nonlocality: here, every space derivative in the matrix element counts as $O(p)$, and spatial components $s^i \sim O(p^{-1})$, so one is not allowed to truncate the Taylor series. One may, however, expand the fermion propagators contained in $S^{(1)}$. In the HBChPT regime, this expansion reads

$$\frac{M_\Psi + \not{p}}{M_\Psi^2 - p^2} = \frac{1 + \not{p}}{-2v \cdot l} + \frac{1}{M_\Psi} \left(\frac{\not{v}}{-2v \cdot l} + \frac{(1 + \not{p})l^2}{4(v \cdot l)^2} \right) + O\left(\frac{1}{M_\Psi^2}\right), \quad (2.33)$$

where $p = M_\Psi v + l$, the unit vector $v = (1, \mathbf{0})$, and as usual $\not{p} = p_\mu \gamma^\mu$. From this expansion one observes that i) each term with an additional power of M_Ψ^{-1} is $O(p)$ suppressed with respect to the previous term, so the series can be truncated at a given chiral order, and ii) each term is a polynomial in 3-momenta. According to the above properties, at a given order in the chiral expansion, $S^{(1)}$ in coordinate space is a finite sum of spatial δ -functions and derivatives thereof. Substituting into Eq. (2.30) and integrating by parts, one finally obtains a finite sum containing matrix elements of the fermion bilinears and derivatives thereof, taken at the same space-time point R . In addition, we recall that the field Ψ obeys the free Dirac equation, so all time derivatives can be eliminated at the end.

The expression which one obtains after performing all Taylor expansions, is much simpler than the original one. Whereas the matrix element in Eq. (2.30) involves a “dynamical” density matrix (with non-coinciding time variables), which is strongly dependent on the description of the structure of the nucleus, after the Taylor expansion one gets terms of the type

$$(\Omega | : \bar{\Psi}_b(0, \mathbf{R}) \overset{\leftrightarrow}{\partial}_{i_1} \cdots \overset{\leftrightarrow}{\partial}_{i_n} \Psi_a(0, \mathbf{R}) : | \Omega), \quad n = 0, 1, \dots, \quad (2.34)$$

which have a simpler interpretation. For example, using the Fierz transformation, the term without derivatives can be related to the matrix elements of local scalar, vector, ... fermion bilinears $(\Omega | : \bar{\Psi} \Gamma_i \Psi : | \Omega)$ with $\Gamma_i = 1, \gamma_\mu, \dots$. To lowest order in the chiral expansion, these matrix elements coincide with the scalar, vector ... formfactors of the nucleus, which can be parametrized, e.g., in terms of the pertinent radii, without any more detailed explicit knowledge of the nuclear structure.

Next, we briefly consider the case of the non-standard counting. As it will be demonstrated below by explicit calculations, the contributions to the self-energy, corresponding to the non-standard counting regime, do not appear up to and including $O(p^5)$ in the pion two-point function in the vicinity of the mass shell. For the general pion kinematics this is not always the case. In particular,

it has been shown [28] that, for a large class of the collective phenomena, the elementary excitations can be described by the non-relativistic dispersion law $E = \gamma \mathbf{p}^2$. Moreover, it has been argued, that the same dispersion law applies in the Goldstone boson condensed phase of the nuclear ground state (see e.g. [29] where this issue is considered in detail). So, in order to be able to use ChPT[®] to study the phenomenon of the pion (and kaon) condensation in general [2, 30, 31, 32], it is necessary to modify counting rules for the external momenta and to assign $E \sim p^2$ to the energy, while 3-momentum counting remains standard [11]. At the next step, one has to look for the pole in the two-point function with the vanishing energy E , which signals that the condensation has indeed occurred. One sees (section 4.2 of Ref. [11]) that in the asymmetric matter where the density exceeds the critical value given by Eq. (4.10) of this paper, the inverse pion propagator indeed develops a pole at $E, \mathbf{p}^2 \rightarrow 0$ and $D = \mathbf{p}^2/E$ finite. This means that for the densities that obey the condition (4.10), the non-standard counting may appear already at the tree level. Note as well, that Eq. (4.10) implies that the density is counted at $O(p^2)$ rather than at $O(p^3)$. Another example, where the non-standard counting appears at the leading order, is the in-medium $\pi\pi$ scattering amplitude [11]. In this paper, we do not consider the non-standard regime in detail since in the context of pionic atoms the contribution from this regime arises at the one-loop level. In this case, our approach should be adapted correspondingly.

To summarize, chiral power counting helps to systematically organize the perturbation expansion in the background of a static nucleus, in terms of *local* distributions. This construction, however, heavily relies on the assumption that only the momentum regime relevant in HBChPT, gives rise to non-zero contributions in the coefficient functions $S^{(n)}$. This conjecture may or may not hold – in the latter case one has the non-standard counting which has to be dealt with separately. Non-local quantities, describing more detailed aspects of nuclear structure, may become necessary here.

In the section 3 we present the explicit calculation of the pion self-energy operator at $O(p^5)$. As already mentioned above, we demonstrate that only the HBChPT momentum regime is relevant in Feynman integrals at this order, so the answer can be given entirely in terms of local distributions. In this paper we do not attempt to analyze higher orders in the chiral expansion.

2.6 The static limit

In this section we examine in some detail the reasoning behind the picture of a static nucleus, and explain the meaning of the “round-bracket” notation which was introduced in Eq. (2.23). As was stated, the procedure involves two steps when evaluating the matrix elements (2.22) in the background of a (heavy) nucleus: first, removing a momentum conserving delta function in the CM frame [which coincides with the nuclear rest frame for an infinitely heavy nucleus], and

secondly, passing to the static nucleus limit in terms of the *velocities* of the initial and final nuclei.

For the sake of illustration, let us first consider the term in the two-point function (2.22), corresponding to the in-vacuum self-energy of the pion:

$$\begin{aligned} & \frac{-i}{(2BF)^2} \int d^4x d^4y e^{iq' \cdot x - iq \cdot y} \langle \Omega(\mathbf{P}') | \frac{\delta^2 S^{(0)}(j)}{\delta p^+(x) \delta p^-(y)} | \Omega(\mathbf{P}) \rangle \Big|_{j=j_0} \\ &= (2\pi)^3 \delta^3(\mathbf{P}' + \mathbf{q}' - \mathbf{P} - \mathbf{q}) \frac{-i}{(2BF)^2} \int d^4x d^4y e^{iq' \cdot x - iq \cdot y} \langle \Omega | \frac{\delta^2 S^{(0)}(j)}{\delta p^+(x) \delta p^-(y)} | \Omega \rangle \Big|_{j=j_0}, \end{aligned} \quad (2.35)$$

with $\langle \Omega | \Omega \rangle = 1$.

Next, consider the term with one medium insertion. Using translational invariance and taking the limit $M_\Omega \rightarrow \infty$, we obtain [cf Eq. (2.30)] in the CM frame:

$$\begin{aligned} & \int d^4x d^4y D_{ba}^{(1)}(q', q | x, y) \langle \Omega(\mathbf{P}') | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega(\mathbf{P}) \rangle \\ &= (2\pi)^3 \delta^3(\mathbf{P}' + \mathbf{q}' - \mathbf{P} - \mathbf{q}) \int d^4x d^4y D_{ba}^{(1)}(q', q | x, y) \langle \Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega \rangle, \end{aligned} \quad (2.36)$$

where

$$\begin{aligned} \langle \Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega \rangle &= \int \frac{d^3\mathbf{Q}}{(2\pi)^3} e^{i\mathbf{Q} \cdot \frac{\mathbf{x}+\mathbf{y}}{2}} f_{ba}(\mathbf{Q}, \mathbf{x} - \mathbf{y}), \\ f_{ba}(\mathbf{Q}, \mathbf{x} - \mathbf{y}) &= \langle \Omega(-\mathbf{L} - \frac{\mathbf{Q}}{2}) | : \bar{\Psi}_b\left(0, \frac{\mathbf{x}-\mathbf{y}}{2}\right) \Psi_a\left(0, -\frac{\mathbf{x}-\mathbf{y}}{2}\right) : | \Omega(-\mathbf{L} + \frac{\mathbf{Q}}{2}) \rangle, \end{aligned} \quad (2.37)$$

and $\mathbf{L} = \frac{1}{2}(\mathbf{q}' + \mathbf{q})$ [Note that in the static limit $M_\Omega \rightarrow \infty$ the bracket matrix element in Eq. (2.37) should not depend on the momentum \mathbf{L} , but only on the momentum transfer \mathbf{Q} since, in this limit, the round-bracket matrix element in the same equation should depend on the vectors \mathbf{x}, \mathbf{y} alone]. From these examples we arrive at the following interpretation. The round-bracket state $|\Omega\rangle$ denotes the static nucleus fixed at a given point of space (at the origin). As mentioned already, this state is assumed to be spin-saturated for simplicity. The expectation value of any operator in the state $|\Omega\rangle$ is translation-invariant in time (but not in space). Furthermore, this expectation value (round-bracket matrix element) is equal to the Fourier transform of a corresponding formfactor evaluated in the Fock space. The generalization for the higher-order terms is straightforward.

Next, we consider in more detail the role of the restriction to spin-saturated states Ω . Let us start with the matrix element of the operator $: \bar{\Psi}(x) \gamma_\mu \Psi(x) :$

In order to pass easily to the static limit, it is advantageous to work in terms of velocities $v_\mu = P_\mu/M_\Omega$, $v'_\mu = P'_\mu/M_\Omega$ instead of momenta P_μ , P'_μ . The above current is conserved since $\Psi(x)$ is a free field. Then, from the assumption that the bound state of the nucleus does not depend on the internal quantum numbers, it follows that the Fock space matrix element of this current is described by a single scalar formfactor:

$$\langle \Omega(\mathbf{P}') | : \bar{\Psi}(0) \gamma_\mu \Psi(0) : | \Omega(\mathbf{P}) \rangle = \frac{1}{2} (v' + v)_\mu F(t), \quad t = (P' - P)^2, \quad (2.38)$$

where $F(t)$ is the form factor of the charge distribution related to the current $\bar{\Psi} \gamma_\mu \Psi$. In the static limit, when $v_\mu, v'_\mu \rightarrow (1, \mathbf{0})$ and $t \rightarrow -(\mathbf{P}' - \mathbf{P})^2 = -(\mathbf{q}' - \mathbf{q})^2$, according to Eq. (2.37), this matrix element reduces to

$$\langle \Omega | : \bar{\Psi}(x) \gamma_\mu \Psi(x) : | \Omega \rangle = g_{\mu 0} \int \frac{d^3 \mathbf{Q}}{(2\pi)^3} e^{i \mathbf{Q} \cdot \mathbf{x}} F(-\mathbf{Q}^2). \quad (2.39)$$

Other fermion bilinears can be considered analogously. For example, symmetry considerations imply that the matrix element of the operator $: \bar{\Psi}(x) \sigma_{\mu\nu} \Psi(x) :$ can involve only the Lorentz-structure $v'_\mu v_\nu - v'_\nu v_\mu$. Then, in the static limit, the round-bracket matrix element should vanish:

$$\langle \Omega | : \bar{\Psi}(x) \sigma_{\mu\nu} \Psi(x) : | \Omega \rangle = 0. \quad (2.40)$$

To summarize, the use of equations of motion and symmetries, together with the simplifying choice of a spin-saturated background nucleus Ω , greatly reduces the number of independent round-bracket matrix elements which serve as an empirical input in the construction of the self-energy operator.

Finally, we mention that for the free Fermi-gas the equality $(\Omega_0 | \Omega_0) = 1$ and further, the equations (2.27) in the rigorous sense are understood as follows

$$\langle \Omega_0 | b^\dagger(\mathbf{l}_1, s_1) b(\mathbf{l}_2, s_2) | \Omega_0 \rangle = (2\pi)^3 2l_1^0 \delta^3(\mathbf{l}_1 - \mathbf{l}_2) \theta(k_F - |\mathbf{l}_1|) \langle \Omega_0 | \Omega_0 \rangle, \quad (2.41)$$

and so forth.

3 Pion self-energy at $O(p^5)$

In this section we demonstrate the general rules formulated in previous sections, by presenting the detailed calculation of the pion self-energy operator at $O(p^5)$ in ChPT in the presence of the finite nucleus. This is an instructive exercise, even though the calculation up to this order yields just the leading terms of the pion-nuclear optical potential, those linear in proton and neutron densities.

3.1 Pion self-energy in the vacuum

The two-point function of the pseudoscalar densities in the vacuum is given by [cf with Eqs. (2.11) and (2.22)]

$$(2\pi)^4 \delta^4(q' - q) D^{(0)}(q^2) = \frac{-i}{(2BF)^2} \int d^4x d^4y e^{iq' \cdot x - iq \cdot y} \frac{\delta^2 S^{(0)}(j)}{\delta p^+(x)\delta p^-(y)} \Big|_{j=j_0}. \quad (3.1)$$

Up to and including $O(p^4)$, only the diagrams shown in Fig. 2 contribute to the self-energy of the pseudoscalar densities in the vacuum. The calculations are most easily done in the so-called σ -model parameterization

$$U = \left(1 - \frac{\boldsymbol{\pi}^2}{F^2}\right)^{1/2} + \frac{i\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{F}, \quad (3.2)$$

where $\boldsymbol{\pi}(x)$ denotes the interpolating pion field. We use this parameterization throughout this paper [of course, the two-point function of the pseudoscalar densities does not depend on the choice of a particular parameterization]. A straightforward calculation with the use of the Lagrangian from appendix A gives

$$D^{(0)}(q^2) = \frac{1}{q^2 - m_\pi^2} + \frac{1}{q^2 - m_\pi^2} \tilde{\Pi}^{(0)}(q^2) \frac{1}{q^2 - m_\pi^2} + \dots, \quad (3.3)$$

where the dots stand for higher-order terms in the chiral expansion, and

$$\tilde{\Pi}^{(0)}(q^2) = (q^2 - m_\pi^2) A^{(0)} + (q^2 - m_\pi^2)^2 B^{(0)}(q^2), \quad (3.4)$$

where $B^{(0)}(q^2)$ is finite at $q^2 \rightarrow m_\pi^2$ and $d \neq 4$. The pertinent contribution to the self-energy is given by

$$\Pi^{(0)}(E; \mathbf{q}', \mathbf{q}) = (2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q}) \tilde{\Pi}^{(0)}(E^2 - \mathbf{q}^2). \quad (3.5)$$

The expression for $A^{(0)}$ takes the form

$$\begin{aligned} A^{(0)} &= \frac{m_\pi^2}{F^2} \left(2l_4^r - \frac{1}{16\pi^2} \ln \frac{m_\pi^2}{\mu^2} \right) + \frac{4m_{\pi^0}^2}{F^2} l_3^r + e^2 [2(3 - \xi) \lambda_{\text{IR}} \right. \\ &\quad \left. - \frac{20}{9} (k_1^r + k_2^r - 2k_5^r - 2k_6^r) + \frac{8}{9} k_7^r + 8k_8^r] + O(d - 4), \end{aligned} \quad (3.6)$$

where μ denotes the scale of the dimensional regularization. Note that we tame both ultraviolet and infrared divergences in this regularization, and we attach the subscript "IR" to

$$\lambda_{\text{IR}} = \frac{\mu^{d-4}}{16\pi^2} \left(\frac{1}{d-4} - \frac{1}{2} (\Gamma'(1) + \ln 4\pi + 1) \right), \quad (3.7)$$

in order to distinguish ultraviolet and infrared divergences. Finally, we note that the explicit expression for $B^{(0)}(p^2)$ is never needed, because it does not contribute

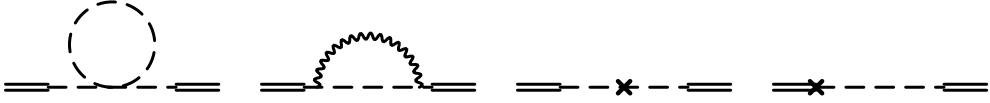


Figure 2: Diagrams contributing to the two-point function of the pseudoscalar densities at $O(p^4)$ in the σ -model parameterization. Double, dashed and wavy lines stand for the pseudoscalar densities, pions and photons, respectively. Crosses denote the vertices with $O(p^4)$ LECs l_i, k_i .

to the residue of the two-point function at this order of the chiral expansion – for this reason we do not display it here.

As one immediately sees from Eq. (3.6), the pion self-energy is both infrared-divergent and gauge-dependent off-shell, i.e. when $q^2 \neq m_\pi^2$. Below we shall demonstrate that in the eigenvalue equation for determining the binding energy of the pion-nuclear bound state, where pion self-energy plays the role of the potential, such off-shell effects begin to contribute only at higher chiral order and can be neglected at the level we are working.

3.2 One medium insertion

According to Eq. (2.10), the contribution with one medium insertion is expressed in terms of the coefficient function (2.31) where, up to and including $O(p^5)$, the quantity $D^{(1)}$ is given by the diagrams in Fig. 3. These are: the one-photon exchange diagram, contributions from the anomalous magnetic moment of the nucleon, the Weinberg-Tomozawa vertex, contributions from the LECs c_i, f_i , and the nucleon pole diagrams

$$(q'^2 - m_\pi^2)(q^2 - m_\pi^2)D_{ba}^{(1)}(q', q|x, y) = \delta^4(x - y)e^{i(q' - q) \cdot x}[D_{ba}^{(1\gamma)}(q', q) + D_{ba}^{(an)}(q', q) + D_{ba}^{(WT)}(q', q) + D_{ba}^{(cf)}(q', q)] + D_{ba}^{(P)}(q', q|x, y), \quad (3.8)$$

where we use the physical value of the squared pion mass, m_π^2 , instead of its $O(p^2)$ value, since the difference is of higher order in ChPT, which we neglect anyway. Further,

$$\begin{aligned} D_{ba}^{(1\gamma)}(q', q) &= e^2(\gamma_\mu)_{\beta\alpha}\frac{1}{2}(1 + \tau^3)_{ji}(q' + q)_\nu D^{\mu\nu}(q' - q), \\ D_{ba}^{(an)}(q', q) &= \frac{e^2}{m}(i\sigma_{\mu\lambda})_{\beta\alpha}\left(\frac{c_6 + 2c_7}{4} + \frac{c_6}{4}\tau^3\right)_{ji}(q' + q)_\nu(q - q')^\lambda D^{\mu\nu}(q' - q), \\ D_{ba}^{(WT)}(q', q) &= -\frac{1}{4F^2}(\gamma_\mu)_{\beta\alpha}(\tau^3)_{ji}(q' + q)^\mu, \\ D_{ba}^{(cf)}(q', q) &= \left(\frac{4c_1m_{\pi^0}^2 - 2c_2q^{0'}q^0 - 2c_3(q'q)}{F^2} + 2e^2f_1\right)\delta_{\beta\alpha}\delta_{ji} \end{aligned}$$

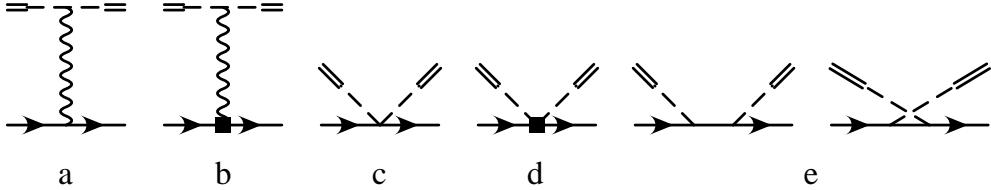


Figure 3: Diagrams contributing to the quantity $D^{(1)}$ up to and including $O(p^5)$: (a) one-photon exchange; (b) contributions from the anomalous magnetic moment of the nucleon; (c) Weinberg-Tomozawa vertex; (d) contributions from LECs c_i, f_i ; (e) nucleon pole diagrams.

$$\begin{aligned}
& + \frac{e^2}{2} f_2 \delta_{\beta\alpha} (\tau^3)_{ji} + \frac{ic_4}{F^2} q'_\nu q_\mu \sigma_{\beta\alpha}^{\mu\nu} \tau^3_{ji}, \\
D_{ba}^{(P)}(q', q | x, y) &= -\frac{g_A^2}{2F^2} (\not{q}' \gamma_5 S_n(x-y) \not{q} \gamma_5)_{\beta\alpha} \frac{1}{2} (1 + \tau^3)_{ji} e^{iq' \cdot x - iq \cdot y} \\
& - \frac{g_A^2}{2F^2} (\not{q} \gamma_5 S_p(x-y) \not{q}' \gamma_5)_{\beta\alpha} \frac{1}{2} (1 - \tau^3)_{ji} e^{iq \cdot y - iq' \cdot x}. \quad (3.9)
\end{aligned}$$

In these formulae, $D^{\mu\nu}$ and $S_{p,n}$ stand for the free photon and baryon propagators, respectively:

$$\begin{aligned}
D^{\mu\nu}(x) &= \int \frac{d^4 l}{(2\pi)^4} e^{-il \cdot x} \frac{1}{l^2} \left(g^{\mu\nu} - (1 - \xi) \frac{l^\mu l^\nu}{l^2} \right), \\
S_i(x) &= \int \frac{d^4 l}{(2\pi)^4} e^{-il \cdot x} \frac{1}{M_i - l}. \quad (3.10)
\end{aligned}$$

The corresponding contribution to the self-energy is

$$2\pi\delta(q'^0 - q^0)\Pi^{(1)}(E; \mathbf{q}', \mathbf{q}) = \int d^4 x d^4 y D_{ba}^{(1)}(q', q | x, y) (\Omega | : \bar{\Psi}_b(x) \Psi_a(y) : | \Omega) \quad (3.11)$$

We proceed to investigate term by term in detail the contributions (3.9) to the two-point function of the pseudoscalar densities.

3.2.1 Coulomb potential and contribution from the anomalous magnetic moment of the nucleon

According to Eq. (3.9), the one photon exchange contribution to the two-point function of the pseudoscalar densities is given by

$$\bar{D}^{(1\gamma)}(q', q) = \frac{1}{q'^2 - m_\pi^2} 2\pi\delta(q'^0 - q^0)\Pi^{(1\gamma)}(E; \mathbf{q}', \mathbf{q}) \frac{1}{q^2 - m_\pi^2}, \quad (3.12)$$

where

$$\begin{aligned} 2\pi\delta(q^{0'} - q^0)\Pi^{(1\gamma)}(E; \mathbf{q}', \mathbf{q}) &= e^2 \int d^4x e^{i(\mathbf{q}' - \mathbf{q}) \cdot \mathbf{x}} (q' + q)_\nu D^{\mu\nu}(q' - q) \times \\ &\quad \times \langle \Omega | : \bar{\Psi}(x) \gamma_\mu \frac{1}{2} (1 + \tau^3) \Psi(x) : | \Omega \rangle. \end{aligned} \quad (3.13)$$

In order to demonstrate that the longitudinal piece depending on the gauge parameter ξ does not contribute in Eq. (3.13), we write this matrix element in the form

$$\langle \Omega | : \bar{\Psi}(x) \gamma_\mu \frac{1}{2} (1 + \tau^3) \Psi(x) : | \Omega \rangle = g_{\mu 0} \frac{1}{2} (\rho^0(\mathbf{x}) + \rho^3(\mathbf{x})), \quad (3.14)$$

with the isoscalar and isovector densities $\rho^0 = \rho_p + \rho_n$ and $\rho^3 = \rho_p - \rho_n$. The density in (3.14) is related to the Fourier-transform of the formfactor of the nuclear charge distribution,

$$\frac{1}{2} (\rho^0(\mathbf{x}) + \rho^3(\mathbf{x})) = \rho_p(\mathbf{x}) = \int \frac{d^3\mathbf{l}}{(2\pi)^3} e^{i\mathbf{l}\cdot\mathbf{x}} F_{0+3}[-\mathbf{l}^2], \quad (3.15)$$

where

$$\langle \Omega'(\mathbf{P}') | : \bar{\Psi}(0) \gamma_\mu \frac{1}{2} (1 + \tau^3) \Psi(0) : | \Omega(\mathbf{P}) \rangle = \frac{1}{2} (v' + v)_\mu F_{0+3}[t] \quad (3.16)$$

with $t = (P'_\Omega - P_\Omega)^2$ [cf Eqs. (2.38) (2.39)]. At the order in ChPT we are working, the formfactor $F_{0+3}[t]$ is proportional to the electromagnetic formfactor of the nucleus Ω

The presence of $\delta(p^0 - q^0)$ in Eq. (3.12) implies that the one-photon exchange contribution turns out to be effectively instantaneous. Eq. (3.13) takes the form

$$\Pi^{(1\gamma)}(E; \mathbf{q}', \mathbf{q}) = -2E \int d^3\mathbf{x} e^{-i(\mathbf{q}' - \mathbf{q}) \cdot \mathbf{x}} V_C(\mathbf{x}), \quad (3.17)$$

where

$$V_C(\mathbf{x}) = \int d^3\mathbf{r} \frac{\alpha}{|\mathbf{x} - \mathbf{r}|} \rho_p(\mathbf{r}), \quad (3.18)$$

and $\alpha = e^2/(4\pi)$ denotes the fine structure constant. Recall that, in the chiral counting, $\rho^{[0,3]}(\mathbf{r}) \sim p^3$ and $e \sim p$. It is straightforward to observe that the quantity $\Pi^{(1\gamma)}$ given by Eq. (3.17), counts as $O(p)$. According to our conventions from section 2 [the actual chiral power of $\Pi(E; \mathbf{q}', \mathbf{q})$ differs by 3 units from the chiral power assigned, in order to stay in conformity with the chiral counting in the vacuum], the Coulomb term contributes to the self-energy at $O(p^4)$.

We finally note that the contribution from the anomalous magnetic moment of the nucleon – which otherwise would contribute at $O(p^5)$ – vanishes since the matrix element of the operator $: \bar{\Psi} \sigma_{\mu\nu} \Psi :$ between the nuclear states disappears in the static limit [see Eq. (2.40)]. Consequently,

$$\Pi^{(an)}(E; \mathbf{q}', \mathbf{q}) = 0. \quad (3.19)$$

3.2.2 Contact contributions

Contact contributions include: the Weinberg-Tomozawa term [$O(p^4)$], and the LEC contributions [$O(p^5)$] involving c_i and f_i . From Eq. (3.9) one obtains

$$\begin{aligned} \Pi^{(WT)+(cf)}(E; \mathbf{q}', \mathbf{q}) &= \int d^3 \mathbf{x} e^{-i(\mathbf{q}' - \mathbf{q}) \cdot \mathbf{x}} [\Pi_0(E; \mathbf{x}) + \mathbf{q}' \cdot \mathbf{q} \Pi_2(E; \mathbf{x})], \\ \Pi_0(E; \mathbf{x}) &= -\frac{E}{2F^2} \rho^3(\mathbf{x}) + \left(\frac{4c_1 m_{\pi^0}^2 - 2(c_2 + c_3)E^2}{F^2} + 2e^2 f_1 \right) \sigma^0(\mathbf{x}) + \frac{e^2}{2} f_2 \sigma^3(\mathbf{x}), \\ \Pi_2(E; \mathbf{x}) &= \frac{2c_3}{F^2} \sigma^0(\mathbf{x}), \end{aligned} \quad (3.20)$$

with the isoscalar and isovector scalar densities

$$\sigma^{[0,3]}(\mathbf{x}) = (\Omega | : \bar{\Psi}(x)[1, \tau^3] \Psi(x) : | \Omega). \quad (3.21)$$

Note that the contribution proportional to c_4 , vanishes for the static nucleus. We would like to stress that, in order to be consistent with the chiral counting in the nuclear matrix elements and our description of the nucleus, one has to count the difference between $\sigma^{[0,3]}(\mathbf{x})$ and $\rho^{[0,3]}(\mathbf{x})$ at a higher chiral order than these quantities themselves. This difference, which involves $1 - \gamma_0$, is determined by the overlap of the “small” components of the Dirac wave function of the nucleon. For a slowly moving nucleon inside the nucleus, this difference is suppressed by a factor p^2 as compared to the overlap of the “large” components. This can be directly seen for the case of the uniform density, where $\rho^{[0,3]} - \sigma^{[0,3]} \sim O(k_F^5)$, whereas each term individually is of order k_F^3 . Below, we shall always use the counting

$$\frac{\rho^{[0,3]}(\mathbf{x}) - \sigma^{[0,3]}(\mathbf{x})}{\rho^{[0,3]}(\mathbf{x})} \sim O(p^2), \quad (3.22)$$

and eliminate $\sigma^{[0,3]}(\mathbf{x})$ in favor of $\rho^{[0,3]}(\mathbf{x})$ in all expressions.

Note also that one may relate the matrix elements (3.21) to the scalar form-factor of the nucleus

$$\begin{aligned} \sigma^0(\mathbf{x}) &= \frac{1}{8ic_1 B} \frac{\delta}{\delta s^0(x)} (\Omega | \hat{S}(j) | \Omega) \Big|_{j=j_0} + \dots, \\ \sigma^3(\mathbf{x}) &= \frac{1}{4ic_5 B} \frac{\delta}{\delta s^3(x)} (\Omega | \hat{S}(j) | \Omega) \Big|_{j=j_0} + \dots, \end{aligned} \quad (3.23)$$

modulo higher-order terms in chiral expansion. At the order we are working, these higher-order terms can be neglected.

3.2.3 Nucleon loop in the presence of the nucleus

The contribution coming from the diagrams of Fig. 3e, involves the matrix element of two fermion fields taken at different space-time points. In order to evaluate this contribution, one has to use the expansion described in sections 2.5, 2.6, along with the bookkeeping based on chiral symmetry. A straightforward, yet tedious calculation leads to:

$$\Pi^{(P)}(E; \mathbf{q}', \mathbf{q}) = \frac{g_A^2}{2F^2} \int d^3 \mathbf{x} e^{-i(\mathbf{q}' - \mathbf{q}) \cdot \mathbf{x}} [\Pi_4(E; \mathbf{q}', \mathbf{q} | \mathbf{x}) + \Pi_5(E; \mathbf{q}', \mathbf{q} | \mathbf{x})] + \dots \quad (3.24)$$

where

$$\begin{aligned} \Pi_4(E; \mathbf{q}', \mathbf{q} | \mathbf{x}) &= \frac{\mathbf{q}' \cdot \mathbf{q}}{E} \rho^3(\mathbf{x}), \\ \Pi_5(E; \mathbf{q}', \mathbf{q} | \mathbf{x}) &= \frac{(M_n - M_p) \mathbf{q}' \cdot \mathbf{q}}{E^2} \rho^3(\mathbf{x}) \\ &+ \left(\frac{E^2 - 2\mathbf{q}' \cdot \mathbf{q}}{2M_N} + \frac{3(\mathbf{q}' \cdot \mathbf{q})^2 - \mathbf{q}'^2 \mathbf{q}^2}{4M_N E^2} \right) \rho^0(\mathbf{x}). \end{aligned} \quad (3.25)$$

Here $M_N = \frac{1}{2}(M_p + M_n)$, and the dots in Eq. (3.24) stand for the higher-order terms in chiral expansion.

3.3 Two medium insertions

The coefficient function $D^{(2)}$, which corresponds to the two medium insertions in the pion self-energy, is given by

$$\begin{aligned} D_{b_1 b_2 a_1 a_2}^{(2)}(q', q | x_1, x_2, y_1, y_2) &= \frac{-i}{(2BF)^2} \int d^4 u d^4 z e^{iq' \cdot u - iq \cdot z} \frac{\delta^2 S_{b_1 b_2 a_1 a_2}^{(2)}(j | x_1, x_2, y_1, y_2)}{\delta p^+(u) \delta p^-(z)} \Big|_{j=j_0}. \end{aligned} \quad (3.26)$$

Up to and including $O(p^5)$, there is a single contribution which stems from the pseudovector pion-nucleon Lagrangian. This contribution, which is diagrammatically shown in Fig 4, equals

$$\begin{aligned} (q'^2 - m_\pi^2)(q^2 - m_\pi^2) D_{b_1 b_2 a_1 a_2}^{(2)}(q', q | x_1, x_2, y_1, y_2) &= \frac{ig_A^2}{16F^2} e^{iq' \cdot x_1 - iq \cdot x_2} \times \\ &\times \delta^4(x_1 - y_1) \delta^4(x_2 - y_2) (\not{q}' \gamma_5)_{\beta_1 \alpha_1} (\tau^1 + i\tau^2)_{j_1 i_1} (\not{q} \gamma_5)_{\beta_2 \alpha_2} (\tau^1 - i\tau^2)_{j_2 i_2} \\ &+ \text{crossed term}. \end{aligned} \quad (3.27)$$

The pertinent contribution to the self-energy is

$$2\pi \delta(q^{0'} - q^0) \Pi^{(2)}(E; \mathbf{q}', \mathbf{q}) = -\frac{ig_A^2}{16F^2} \int d^4 x d^4 y e^{iq' \cdot x - iq \cdot y} \times$$



Figure 4: Two medium insertions: the diagrams contributing to the quantity $D^{(2)}$, Eq. (3.27).

$$\times \langle \Omega | : \bar{\Psi}(x) \not{d} \gamma_5(\tau^1 + i\tau^2) \Psi(x) \bar{\Psi}(y) \not{d} \gamma_5(\tau^1 - i\tau^2) \Psi(y) : | \Omega \rangle + \text{crossed term}. \quad (3.28)$$

It is convenient to work in the momentum representation for the free fields Ψ . For the self-energy, one gets the following expression

$$\begin{aligned} 2\pi\delta(q^{0'} - q^0)\Pi^{(2)}(E; \mathbf{q}', \mathbf{q}) &= \frac{ig_A^2}{16F^2} \sum_{s_1 s_2 s_3 s_4} \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \theta(l_1^0)\delta(l_1^2 - M_p^2) \times \\ &\times \theta(l_1^0 + q^{0'})\delta((l_1 + q')^2 - M_n^2)\theta(l_2^0)\delta(l_2^2 - M_n^2)\theta(l_2^0 - q^0)\delta((l_2 - q)^2 - M_p^2) \times \\ &\times \bar{u}(\mathbf{l}_1, s_1) \not{d} \gamma_5 u(\mathbf{l}_1 + q', s_3) \bar{u}(\mathbf{l}_2, s_2) \not{d} \gamma_5 u(\mathbf{l}_2 - q, s_4) \times \\ &\times \langle \Omega | b^\dagger(\mathbf{l}_1 s_1) b^\dagger(\mathbf{l}_2 s_2) b(\mathbf{l}_3 s_3) b(\mathbf{l}_4 s_4) | \Omega \rangle + \dots, \end{aligned} \quad (3.29)$$

where $u(\mathbf{l}s) = \begin{pmatrix} u_p(\mathbf{l}s) \\ u_n(\mathbf{l}s) \end{pmatrix}$ denotes the 8-component Dirac spinor of the nucleon, and the dots indicate terms with creation/annihilation operators for antinucleons, and the crossed term. Below, we shall demonstrate that, in analogy with the infinite nuclear medium [11], the above contribution vanishes (terms with antinucleons can be considered similarly). The reason for this lies in the fact that we still imply a soft cutoff on the 3-momenta of the nucleons within the state Ω [albeit with the position-dependent ‘‘local Fermi momentum’’ k_F]. On the other hand, the argument of the δ -functions in Eq. (3.29) does not vanish in the soft momentum region. For example, the solution for $|\mathbf{l}_1|$ is

$$|\mathbf{l}_1| = \frac{-\Delta|\mathbf{q}'|\cos\theta \pm \sqrt{\Delta^2|\mathbf{q}'|^2\cos^2\theta - (M_p^2 q_0'^2 - \Delta^2)(q_0'^2 - |\mathbf{q}'|^2\cos^2\theta)}}{q_0'^2 - |\mathbf{q}'|^2\cos^2\theta}, \quad (3.30)$$

where θ is the angle between \mathbf{q}' and \mathbf{l}_1 , and

$$\Delta = \frac{q_0'^2 - |\mathbf{q}'|^2 + M_p^2 - M_n^2}{2}. \quad (3.31)$$

The quantity Δ is of order p^2 in chiral counting. It is easy to see that, whenever the solution of Eq. (3.30) exists, the quantity $|\mathbf{l}_1|$ counts at chiral order 1. For this reason, the quantity $\Pi^{(2)}(E; \mathbf{q}', \mathbf{q})$ vanishes at $O(p^5)$ in ChPT.

4 Equation for the pion-nucleus bound states

From equations (2.24) and (2.25) one can define the pion scattering amplitude $T(E, \mathbf{q}', \mathbf{q})$ in the presence of the nucleus

$$D(E, \mathbf{q}', \mathbf{q}) = \frac{(2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q})}{E^2 - \mathbf{q}^2 - m_\pi^2} + \frac{1}{E^2 - \mathbf{q}'^2 - m_\pi^2} T(E, \mathbf{q}', \mathbf{q}) \frac{1}{E^2 - \mathbf{q}^2 - m_\pi^2}, \quad (4.1)$$

which obeys the Lippmann-Schwinger equation

$$T(E; \mathbf{q}', \mathbf{q}) = \Pi(E, \mathbf{q}', \mathbf{q}) + \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Pi(E, \mathbf{q}', \mathbf{k}) \frac{1}{E^2 - \mathbf{k}^2 - m_\pi^2} T(E, \mathbf{k}, \mathbf{q}), \quad (4.2)$$

with the self-energy $\Pi(E; \mathbf{q}', \mathbf{q})$ playing the role of the potential. Up to and including $O(p^5)$, this quantity is given by

$$\begin{aligned} \Pi(E; \mathbf{q}', \mathbf{q}) &= (2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q}) \tilde{\Pi}^{(0)}(E^2 - \mathbf{q}^2) + \Pi^{(1)}(E; \mathbf{q}', \mathbf{q}), \\ \Pi^{(1)}(E; \mathbf{q}', \mathbf{q}) &= \Pi^{(1\gamma)} + \Pi^{(an)} + \Pi^{(WT)} + \Pi^{(cf)} + \Pi^{(P)}, \end{aligned} \quad (4.3)$$

where the individual terms of Eq. (4.3) are given in Eqs. (3.4), (3.17), (3.19), (3.20) and (3.24), respectively.

The quantity $\tilde{\Pi}^{(0)}(E^2 - \mathbf{q}^2)$ defined by Eq. (3.4) is both infrared-divergent and gauge-dependent. Below we shall demonstrate that, despite this fact, the position of the bound-state pole in the scattering amplitude $T(E; \mathbf{q}', \mathbf{q})$ is – at this order in chiral expansion – both infrared-finite and gauge-independent. In order to prove this, we note that, using the theory of scattering on two potentials, the scattering amplitude can be given as

$$\begin{aligned} T(E; \mathbf{q}', \mathbf{q}) &= (2\pi)^3 \delta^3(\mathbf{q}' - \mathbf{q}) \nu(E^2 - \mathbf{q}^2) \\ &\quad + \kappa^{1/2}(E^2 - \mathbf{q}'^2) \tau(E; \mathbf{q}', \mathbf{q}) \kappa^{1/2}(E^2 - \mathbf{q}^2), \\ \kappa(s) &= [1 - A^{(0)} - (s - m_\pi^2) B^{(0)}(s)]^{-1}, \quad \nu(s) = (m_\pi^2 - s)[1 - \kappa(s)], \end{aligned} \quad (4.4)$$

where $A^{(0)}$, $B^{(0)}$ are defined in Eq. (3.4), and $\tau(E; \mathbf{q}', \mathbf{q})$ obeys to the following Lippmann-Schwinger equation

$$\begin{aligned} \tau(E; \mathbf{q}', \mathbf{q}) &= U(E, \mathbf{q}', \mathbf{q}) + \int \frac{d^3 \mathbf{k}}{(2\pi)^3} U(E, \mathbf{q}', \mathbf{k}) \frac{1}{E^2 - \mathbf{k}^2 - m_\pi^2} \tau(E, \mathbf{k}, \mathbf{q}), \\ U(E, \mathbf{q}', \mathbf{k}) &= \kappa^{1/2}(E^2 - \mathbf{q}'^2) \Pi^{(1)}(E; \mathbf{q}', \mathbf{q}) \kappa^{1/2}(E^2 - \mathbf{q}^2). \end{aligned} \quad (4.5)$$

From equation (4.4) it is clear that the position of the poles in $T(E; \mathbf{q}', \mathbf{q})$ and $\tau(E; \mathbf{q}', \mathbf{q})$ coincide. On the other hand, the difference between the quantities $\Pi^{(1)}(E; \mathbf{q}', \mathbf{q})$ and $U(E; \mathbf{q}', \mathbf{q})$ which is due to the factors $\kappa^{1/2}$, starts at $O(p^6)$ and can be neglected. This is the statement which we aimed to prove: the pole

position is determined from the equation (4.5) whose potential $U(E; \mathbf{q}', \mathbf{q})$ is infrared-finite and gauge-independent up to and including $O(p^5)$ in ChPT. Note that, since the potential is Hermitian at this order, the position of the bound-state pole can also be found by solving the equivalent Klein-Gordon equation

$$[E^2 - \mathbf{q}'^2 - m_\pi^2] \Phi_E(\mathbf{q}') = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} U(E; \mathbf{q}', \mathbf{q}) \Phi_E(\mathbf{q}), \quad (4.6)$$

where $\Phi_E(\mathbf{q})$ stands for the wave function of the bound state in the momentum representation.

We finally collect all terms of the potential at this order (replacing $\sigma^{[0,3]}(\mathbf{x})$ by $\rho^{[0,3]}(\mathbf{x})$, see Eq. (3.22)):

$$\begin{aligned} U(E; \mathbf{q}', \mathbf{q}) &= \int d^3 \mathbf{x} e^{-i(\mathbf{q}' - \mathbf{q}) \cdot \mathbf{x}} [\tilde{U}(E; \mathbf{q}', \mathbf{q}; \mathbf{x}) + O(p^6)], \\ \tilde{U}(E; \mathbf{q}', \mathbf{q}; \mathbf{x}) &= - \int d^3 \mathbf{r} \frac{\alpha E}{|\mathbf{x} - \mathbf{r}|} [\rho^0(\mathbf{r}) + \rho^3(\mathbf{r})] \\ &- \frac{E - e^2 F^2 f_2}{2F^2} \rho^3(\mathbf{x}) + \frac{4c_1 m_{\pi^0}^2 - 2(c_2 + c_3)E^2 + 2c_3 \mathbf{q}' \cdot \mathbf{q} + 2e^2 F^2 f_1}{F^2} \rho^0(\mathbf{x}) \\ &+ \frac{g_A^2}{2F^2} \left\{ \left[\frac{E^2 - 2\mathbf{q}' \cdot \mathbf{q}}{2M_N} + \frac{3(\mathbf{q}' \cdot \mathbf{q})^2 - \mathbf{q}'^2 \mathbf{q}^2}{4M_N E^2} \right] \rho^0(\mathbf{x}) \right. \\ &\left. + \frac{\mathbf{q}' \cdot \mathbf{q}}{E} \left(1 + \frac{M_n - M_p}{E} \right) \rho^3(\mathbf{x}) \right\}. \end{aligned} \quad (4.8)$$

Equation (4.8) represents our main result: the complete expression of the optical potential for pion scattering on the finite nucleus up to and including $O(p^5)$ in ChPT in the presence of electromagnetic interactions and strong isospin-breaking effects. Note that, unlike most of previous descriptions, the framework described in the present paper enables one to unambiguously obtain the explicit dependence of the optical potential on the momenta (off-shell) \mathbf{q} and \mathbf{q}' . For example, the terms proportional to $(\mathbf{q}' \cdot \mathbf{q})^2 - \mathbf{q}'^2 \mathbf{q}^2$ vanish in the infinite medium. They cannot be obtained from an extrapolation of standard in-medium ChPT or semi-phenomenological approaches starting from pion scattering in nuclear matter.

5 Comparison with existing approaches

In this section, we compare our results

- i)-iii) to the existing calculations of the in-medium pion mass shift in ChPT [9, 11, 12];
- iv) to the pion-nucleus optical potential obtained in Ref. [13] from ChPT;

- v) to the empirical pion-nucleus optical potentials which were widely used in the literature to describe deeply bound states of pions on the heavy nuclei [4].
- i) In order to compare with the calculations performed in the infinite nuclear medium, we put $\rho^0(\mathbf{x}) = \rho_p + \rho_n$, $\rho^3(\mathbf{x}) = \rho_p - \rho_n$, where $\rho_{p,n} = \frac{1}{3\pi^2} (k_F^{(p,n)})^3$ and $k_F^{(p,n)}$ stands for the Fermi-momentum of the proton (neutron). In the limit of a uniform medium, our expression for the self-energy in the absence of electromagnetic interactions ($e = 0$) coincides with the expression for the same quantity at $O(p^5)$, which is given in Eq. (4.5) of Ref. [11], except for the term proportional to $g_A^2(M_n - M_p)$ (or, equivalently, $g_A^2 c_5 B(m_d - m_u)$ in the chiral expansion). Note that, since $e = 0$ is assumed in Ref. [11], at $O(p^5)$ there is no difference whether the LEC c_1 is multiplied by m_π^2 (as in [11]) or $m_{\pi^0}^2$ (as in the present paper). For the same reason, the contribution involving the LECs f_1 , f_2 is not present in Eq. (4.5) of Ref. [11].
- ii) In Ref. [9] the threshold self-energy ($E = m_\pi$) is evaluated at $O(p^6)$ in ChPT, at vanishing 3-momentum, and with $e = 0$, $m_u = m_d$, but at $k_F^{(p)} \neq k_F^{(n)}$. The result, given by Eqs. (4)-(5) of this paper, does not depend on the energy E as well. Up to and including order p^5 , this result agrees (apart from the terms proportional to e^2 and $m_d - m_u$) with our result, if one sets $E^2 = m_\pi^2$. However, ref. [9] does include very important double scattering contributions which first appear at $O(p^6)$.
- iii) The results obtained in Ref. [12] are similar to those of Ref. [9], with the exception that in Ref. [12] the restriction $E^2 = m_\pi^2$ is removed. As was mentioned above, both [9, 12] determine the pole position from the two-point function of the interpolating pion fields. In this quantity one encounters the so-called off-shell ambiguity for $E^2 \neq m_\pi^2$, which should disappear in the calculation of the pole position in order to be compatible with general principles of quantum field theory. In Ref. [12] it is demonstrated that this actually happens if one expands the pole position in powers of the quark mass and eliminates the contributions at $O(p^7)$ and higher (see also [33]).

Concerning numerical studies of the in-medium pion mass shift, the perturbative estimates in all three papers [9, 11, 12] yield results around 10 MeV, which is too small as compared to the “empirically” deduced shift ~ 25 MeV. In Ref. [11] it is shown, that making the partial resummation of the higher-order terms, it is possible to obtain the result for the mass shift which differs from the “perturbative” solution by almost a factor of 2. The main reason for this difference is the strong energy-dependence of the self-energy operator. It should be noted, of course, that the concept of a pion mass shift in nuclear matter, while of some theoretical interest, is of only limited relevance in the context of Coulomb-bound

pionic atom states. It is in fact more satisfactory to directly test the theoretical predictions for the bound-state observables on the experiment by solving the wave equation self-consistently for the finite system – e.g., as done in Ref. [13].

iv) In order to compare with the pion-nucleus optical potential obtained in Ref. [13] by extrapolating the results of in-medium ChPT to the finite nuclei, we rewrite Eq. (4.7) as follows:

$$\begin{aligned}\tilde{U}(E; \mathbf{q}', \mathbf{q}; \mathbf{x}) &= U_C(E; \mathbf{x}) + U_S(E; \mathbf{x}) - (\mathbf{q}' \cdot \mathbf{q}) U_P(E; \mathbf{x}) \\ &\quad + (3(\mathbf{q}' \cdot \mathbf{q})^2 - \mathbf{q}'^2 \mathbf{q}^2) U_D(E; \mathbf{x}),\end{aligned}\tag{5.1}$$

where $U_C(E; \mathbf{x})$ is the Coulomb term (first line of Eq. (4.8)), and $S-$, $P-$, $D-$ wave parts are given by

$$\begin{aligned}U_S(E; \mathbf{x}) &= -\frac{E - e^2 F^2 f_2}{2F^2} \rho^3(\mathbf{x}) \\ &\quad + \frac{4c_1 m_{\pi}^2 - 2(c_2 + c_3 - g_A^2/8M_N)E^2 + 2e^2 F^2 f_1}{F^2} \rho^0(\mathbf{x}) \\ U_P(E; \mathbf{x}) &= -\frac{2(c_3 - g_A^2/4M_N)}{F^2} \rho^0(\mathbf{x}) + \frac{g_A^2}{2F^2 E} \left[1 + \frac{M_n - M_p}{E} \right] \rho^3(\mathbf{x}), \\ U_D(E; \mathbf{x}) &= \frac{g_A^2}{8F^2 M_N E^2} \rho^0(\mathbf{x}).\end{aligned}\tag{5.2}$$

- The S -wave part of the optical potential coincides with the one of Ref. [13] at $O(p^5)$ in ChPT when setting $e = 0$. The low-energy constant c_1 is related to the pion-nucleon sigma term, $\sigma_N = -4c_1 m_{\pi}^2$. The combination $c_2 + c_3$ receives an important contribution from the Δ resonance in the P -wave πN amplitude. The values of these LEC's are individually much larger than their combination appearing in $U_S(m_{\pi}; \mathbf{x})$: they must conspire in just such a way as to reproduce the observed approximate vanishing of the isospin-even πN amplitude at threshold. A small departure from this subtle balance at threshold is expected to have a large impact on pionic bound state energies. This is pointed out in Ref. [13] where it is argued that a large part of the “missing S -wave repulsion” observed in pionic atoms can be explained by the strong energy dependence of the low-energy πN amplitude in ChPT, while the proper gauge-invariant inclusion of the Coulomb potential accounts for most of the remaining effect. This calculation also includes the important double scattering terms of $O(p^6)$. The result, concerning the energy-dependence at $O(p^5)$, is consistent with the findings of Ref. [11] for the infinite medium.

Finally we note that the electromagnetic corrections represented by the LECs f_1 and f_2 were so far not taken into account in the literature.

- For the P -wave part of the optical potential, the authors of Ref. [13] use the time-honored phenomenological parameterization which systematically reproduces a large amount of pionic atom data. This parameterization includes effects of higher order in density, such as the Lorentz-Lorentz correction and pion absorption [1, 2], which cannot be handled within the one-nucleon sector of ChPT. On the other hand, ChPT[®] makes rigorous leading-order statements concerning the P -wave pion self energy for finite systems, and it is at this level where a comparison with phenomenology is useful. In particular $U_P(E; \mathbf{x})$ can be compared to the leading (linear in density) term of the phenomenological parameterization, represented by the πN scattering volumes \tilde{c}_0 and \tilde{c}_1 [1, 2]. (The scattering volumes are usually denoted by c_0 and c_1 , but we need to avoid confusion with the LECs c_1, \dots, c_3 .) One may replace $E \rightarrow m_\pi$ in our expression, and rewrite the P -wave part of the optical potential as

$$\frac{1}{4\pi} \left(1 + \frac{m_\pi}{M_N} \right) U_P(m_\pi; \mathbf{x}) = \tilde{c}_0 \rho^0(\mathbf{x}) - \tilde{c}_1 \rho^3(\mathbf{x}). \quad (5.3)$$

Using the empirical value $\tilde{c}_0 = 0.21 m_\pi^{-3}$, from Table 6.2 of Ref. [2], $g_A = 1.267$ and $F = 92.4$ MeV, we obtain $c_3 = -3.2$ GeV $^{-1}$. There is a theoretical uncertainty in this number, given by the typical size of next-to-leading order corrections. As a consequence, our leading-order estimate of the LEC c_3 in the low-energy πN effective Lagrangian is not incompatible with the value $c_3 = -4.7$ GeV $^{-1}$ quoted in Ref. [11]. As for the isovector term, we find $\tilde{c}_1 = 0.17 m_\pi^{-3}$ at $E = m_\pi$, in agreement with the value used in the set A of Ref. [2].

Note that the relation between the LEC c_3 and the phenomenological P -wave scattering volume \tilde{c}_0 is different in the case of scattering on a single nucleon. Indeed, it is well known that the contribution from the Born (nucleon pole) diagrams to this quantity is zero up to and including terms of $O(1/M_N)$ [34]. The differences between the two cases originate in recoil effects which differ for a single nucleon and for the nucleus as a whole. In the case of scattering on a nucleon the recoil is an effect of order $O(p)$. In the case of a nucleus it is suppressed by an additional factor of $1/A$. It is therefore natural to find such differences already in our $O(p^5)$ calculation. These terms are corrections to the low-density theorem relating the pion self-energy to the forward πN scattering amplitude. It is not possible to identify such corrections in the conventional in-medium ChPT, since the restriction to $\mathbf{p} = \mathbf{q}$ makes the separation between P -wave and S -wave ambiguous.

- There is no D -wave part in the optical potential of Ref. [13]. In our approach, this contribution is partly due to the finite-size effect. As already mentioned, the presence of such terms cannot be reliably established from

the extrapolation of the result obtained at uniform density. They can only be controlled in the consistent approach described in this paper.

- v) The phenomenological optical potential for the pion-nucleus interaction close to threshold is commonly written in coordinate space as [4]

$$2m_\pi \tilde{U}^{\text{phen}} = q(r) + \nabla\alpha(r)\nabla, \quad (5.4)$$

where $q(r)$ and $\alpha(r)$ are taken to be energy-independent. From Eq. (5.2) we observe that there are more terms in the result obtained within ChPT: for example, we have in addition the D -wave part of the potential which is absent in Eq. (5.4). What is important, the counterparts of $q(r)$, $\alpha(r)$ in ChPT depend also explicitly on the energy E . This energy dependence stems from the underlying chiral pion-nucleon dynamics, and is uniquely determined in ChPT.

A few final remarks are in order. First, we wish to mention that in Refs. [4, 9, 11, 12, 13] the terms with $e \neq 0$, $m_d - m_u \neq 0$ in the non-Coulomb part of the optical potential have always been neglected. In this paper, we have included them on the same footing as all other (strong) terms that arise at the same chiral order. Note that the additional electromagnetic terms have sometimes led to significant corrections in the observables bound states of hadrons (see e.g. [17, 35]).

As already mentioned, the optical potentials given in Refs. [4, 13] include additional terms, non-linear in the baryon density. It is well known that these terms give important contributions to the binding energies and widths of pionic atoms. In our counting all these terms start at $O(p^6)$ and have not (yet) been considered in the present paper. Clearly, in order to achieve a good description of pionic atoms, one has to carry out the calculations, outlined in this paper, at least up to and including $O(p^6)$.

The focus in this paper is on the presentation of the ChPT[®] framework for finite systems. Numerical algorithms for solving the bound-state equation and finding the systematic ChPT expansion of the pole positions are a different issue which deserves a separate study. Such an investigation is highly non-trivial. It requires checking the convergence of the chiral expansion for the eigenvalues and examining the stability of these eigenvalues with respect to small variations of the self-energy operator, bearing in mind the strong energy dependence discussed previously.

6 Conclusions

- i) In this paper we propose a novel approach to construct ChPT in the background of a *finite* nucleus: “Chiral Perturbation Theory for Heavy Nuclei”

(ChPT[®]). It develops the rules for systematically evaluating the pion-nucleus optical potential directly for a finite size system. In the present paper the full result of the calculations at $O(p^5)$ is presented.

- ii) The approach is based on certain approximations concerning the description of the nucleus containing A nucleons, where A is a large number. The content of these approximations, given by Eq. (2.21), is as follows: the nucleus in ChPT[®] does not contain free pions and photons. Secondly, the nucleus is not excited by the external pion bound in the atom.
- iii) For simplicity, it is also assumed that the nucleus is static and in a spin-saturated ground state. These assumptions are purely technical and – if needed – can be released at a later stage.
- iv) In ChPT[®], the scattering amplitude of the pion in the nuclear background is given by a sum of terms, each of which is a product of two factors: the scattering amplitude of the pion on $0, 1, 2, \dots$ nucleons which is systematically evaluated in ChPT, and the matrix elements of the normal products of the free fermionic fields between the nuclear states. These (and only these) matrix elements summarize the necessary nuclear structure information.
- v) A crucial ingredient of our approach is the chiral counting for the nuclear matrix elements. The bookkeeping based on chiral symmetry allows one to carry out a Taylor expansion of these matrix elements. This expansion drastically reduces the sensitivity of the final results on the nuclear input.
- vi) The approach can be pursued systematically to higher chiral orders:

First, the chiral counting allows one to unambiguously organize different contributions, and off-shell ambiguities are absent from the beginning.

There are no ultraviolet divergences arising at any step, which would signal an internal inconsistency: the coefficient functions $S^{(n)}$ are finite by construction, and the phenomenological input on the nuclear matrix elements does not introduce ultraviolet divergences either.

The only general property of the theory which might be lost in the approximations that have led to ChPT[®], is the gauge invariance of the bound-state energy at high chiral orders. We have checked gauge invariance explicitly at $O(p^5)$. However, at the present stage, we are not aware of a general proof that this can be done in all orders. Explicit gauge dependence appearing at $O(p^x)$ would signal that, starting from this order, one cannot neglect components of the nuclear wave function that contain pions and photons. From the phenomenological point of view, the situation is harmless.

- vii) Our approach is the generalization of in-medium ChPT which is based on the Fermi-gas model. In the limit of uniform density, our approach reduces to the conventional in-medium ChPT.
- viii) In this paper we present a systematic derivation of the leading-order pion-nucleus optical potential in ChPT, including isospin-breaking contributions, and provide the detailed comparison to other results existing in the literature. It is demonstrated that the present approach generates more terms, with higher spatial derivatives, than conventional models using the local density approximation. In particular, this concerns an additional contribution in the P -wave scattering volume, as well as a D -wave term which is absent in the conventional parameterizations and can be interpreted as a genuine finite-size effect. The negligence of isospin-breaking contributions cannot be justified since these come at the same chiral order as the strong contributions, and since one anyway includes some of the electromagnetic contributions – e.g. the Coulomb potential. An accurate fit to the measured bound-state energies of pionic atoms might help to set bounds on the value of the electromagnetic low-energy constant f_1 which enters the expression of the optical potential at $O(p^5)$.
- ix) Carrying out complete calculations at $O(p^6)$ is challenging from several points of view. First of all, the contributions at this order, including the contributions which are quadratic in the baryon density, are known to be important. We do not give any numerical estimates in this paper, based on the $O(p^5)$ calculations alone. We plan to address $O(p^6)$ calculations, as well as the thorough numerical analysis of the bound-state problem, in future publications.

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A Chiral Lagrangians

For convenience, in this appendix we collect chiral Lagrangians, which are used in the calculation of the pion self-energy at $O(p^5)$. Notations and conventions are identical to the used in Ref. [17], from which the formulae below are taken

$$\begin{aligned} \mathcal{L}_\gamma + \mathcal{L}_\pi^{(p^2)} + \mathcal{L}_\pi^{(e^2)} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \\ &+ \frac{F^2}{4} \langle d^\mu U^\dagger d_\mu U + \chi^\dagger U + U^\dagger \chi \rangle + Z F^4 \langle \mathcal{Q} U \mathcal{Q} U^\dagger \rangle, \end{aligned}$$

$$\begin{aligned}
\mathcal{L}_\pi^{(p^4)} &= \sum_{i=1}^7 l_i O_i^{(p^4)}, & \mathcal{L}_\pi^{(e^2 p^2)} &= F^2 \sum_{i=1}^{10} k_i O_i^{(e^2 p^2)}, \\
\mathcal{L}_N^{(p)} &= \bar{\psi}(i \not{D} - m + \frac{1}{2} g_A \not{\mu} \gamma_5) \psi, \\
\mathcal{L}_N^{(p^2)} &= \sum_{i=1}^7 c_i \bar{\psi} O_i^{(p^2)} \psi, & \mathcal{L}_N^{(e^2)} &= F^2 \sum_{i=1}^3 f_i \bar{\psi} O_i^{(e^2)} \psi. \tag{A1}
\end{aligned}$$

Here, ξ denotes the gauge parameter ($\xi = 1$ in the Feynman gauge), and $\langle \langle \dots \rangle \rangle$ stands for the trace over the isospin indices. The building blocks for constructing the Lagrangian are

$$\begin{aligned}
d_\mu U &= \partial_\mu U - i \mathcal{R}_\mu U + i U \mathcal{L}_\mu, & \begin{pmatrix} \mathcal{R}_\mu \\ \mathcal{L}_\mu \end{pmatrix} &= v_\mu \pm a_\mu + \mathcal{Q} A_\mu, \\
\mathcal{R}_{\mu\nu} &= \partial_\mu \mathcal{R}_\nu - \partial_\nu \mathcal{R}_\mu - i [\mathcal{R}_\mu, \mathcal{R}_\nu], & \mathcal{L}_{\mu\nu} &= \partial_\mu \mathcal{L}_\nu - \partial_\nu \mathcal{L}_\mu - i [\mathcal{L}_\mu, \mathcal{L}_\nu], \\
\chi &= 2B(s + ip), & d_\mu \chi &= \partial_\mu \chi - i \mathcal{R}_\mu \chi + i \chi \mathcal{L}_\mu, & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
\mathcal{C}_R^\mu &= -i [\mathcal{R}_\mu, \mathcal{Q}], & \mathcal{C}_L^\mu &= -i [\mathcal{L}_\mu, \mathcal{Q}], & \mathcal{Q} &= e \text{diag}(\frac{2}{3}, -\frac{1}{3}), \tag{A2}
\end{aligned}$$

in the meson sector, and

$$\begin{aligned}
D_\mu &= \partial_\mu + \Gamma_\mu, & U &= u^2, & u_\mu &= i u^\dagger d_\mu U u^\dagger, \\
\Gamma_\mu &= \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger R_\mu u - \frac{i}{2} u L_\mu u^\dagger, \\
\chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, & \hat{\chi}_+ &= \chi_+ - \frac{1}{2} \langle \chi_+ \rangle, \\
F_{\mu\nu}^\pm &= u^\dagger R_{\mu\nu} u \pm u L_{\mu\nu} u^\dagger, & \hat{F}_{\mu\nu}^+ &= F_{\mu\nu}^+ - \frac{1}{2} \langle F_{\mu\nu}^+ \rangle, \\
Q_\pm &= \frac{1}{2} (u Q u^\dagger \pm u^\dagger Q u), & \hat{Q}_\pm &= Q_\pm - \frac{1}{2} \langle Q_\pm \rangle, & Q &= e \text{diag}(1, 0), \tag{A3}
\end{aligned}$$

in the nucleon sector. Further, $R_\mu, L_\mu, R_{\mu\nu}, L_{\mu\nu}$ are defined just like their pionic counterparts $\mathcal{R}_\mu, \mathcal{L}_\mu, \mathcal{R}_{\mu\nu}, \mathcal{L}_{\mu\nu}$ respectively, with \mathcal{Q} replaced by Q . As usual, $j \doteq \{s, p, v_\mu, a_\mu\}$ denote external scalar, pseudoscalar, vector and axial fields, in the form of 2×2 matrices. In all expressions we drop the terms which do not contain pion fields, and terms of order e^4 .

In the above formulae, the symbol e stands for the electric charge. The quantities F , m and g_A are the pion decay constant, nucleon mass and the nucleon axial constant in the chiral limit. The quantity B is related to the quark condensate in a standard manner, and the quantity Z is expressed through the charged-neutral pion mass difference in the chiral limit

$$m_\pi^2 - m_{\pi^0}^2 = 2e^2 F^2 Z + \dots \tag{A4}$$

Table 1: Operator basis and the divergent parts (in the Feynman gauge) of the LECs in the $O(p^4)$ meson Lagrangian [36] and $O(e^2 p^2)$ meson Lagrangian [37]. The terms that do not contain pion fields, and terms of order e^4 are not displayed.

| i | $O_i^{(p^4)}$ | γ_i |
|-----|--|----------------|
| 1 | $\frac{1}{4} \langle d^\mu U^\dagger d_\mu U \rangle^2$ | $\frac{1}{3}$ |
| 2 | $\frac{1}{4} \langle d^\mu U^\dagger d^\nu U \rangle \langle d_\mu U^\dagger d_\nu U \rangle$ | $\frac{2}{3}$ |
| 3 | $\frac{1}{16} \langle \chi^\dagger U + U^\dagger \chi \rangle^2$ | $-\frac{1}{2}$ |
| 4 | $\frac{1}{4} \langle d^\mu U^\dagger d_\mu \chi + d^\mu \chi^\dagger d_\mu U \rangle$ | 2 |
| 5 | $\langle \mathcal{R}^{\mu\nu} U \mathcal{L}_{\mu\nu} U^\dagger \rangle$ | $-\frac{1}{6}$ |
| 6 | $\frac{i}{2} \langle \mathcal{R}^{\mu\nu} d_\mu U d_\nu U^\dagger$ $+ \mathcal{L}^{\mu\nu} d_\mu U^\dagger d_\nu U \rangle$ | $-\frac{1}{3}$ |
| 7 | $-\frac{1}{16} \langle \chi^\dagger U - U^\dagger \chi \rangle^2$ | 0 |

| i | $O_i^{(e^2 p^2)}$ | σ_i |
|-----|--|----------------------------------|
| 1 | $\langle d^\mu U^\dagger d_\mu U \rangle \langle \mathcal{Q}^2 \rangle$ | $-\frac{27}{20} - \frac{1}{5} Z$ |
| 2 | $\langle d^\mu U^\dagger d_\mu U \rangle \langle \mathcal{Q} U \mathcal{Q} U^\dagger \rangle$ | $2Z$ |
| 3 | $\langle d^\mu U^\dagger \mathcal{Q} U \rangle \langle d_\mu U^\dagger \mathcal{Q} U \rangle$ $+ \langle d^\mu U \mathcal{Q} U^\dagger \rangle \langle d_\mu U \mathcal{Q} U^\dagger \rangle$ | $-\frac{3}{4}$ |
| 4 | $\langle d^\mu U^\dagger \mathcal{Q} U \rangle \langle d_\mu U \mathcal{Q} U^\dagger \rangle$ | $2Z$ |
| 5 | $\langle \chi^\dagger U + U^\dagger \chi \rangle \langle \mathcal{Q}^2 \rangle$ | $-\frac{1}{4} - \frac{1}{5} Z$ |
| 6 | $\langle \chi^\dagger U + U^\dagger \chi \rangle \langle \mathcal{Q} U \mathcal{Q} U^\dagger \rangle$ | $\frac{1}{4} + 2Z$ |
| 7 | $\langle (\chi U^\dagger + U \chi^\dagger) \mathcal{Q}$ $+ (\chi^\dagger U + U^\dagger \chi) \mathcal{Q} \rangle \langle \mathcal{Q} \rangle$ | 0 |
| 8 | $\langle (\chi U^\dagger - U \chi^\dagger) \mathcal{Q} U \mathcal{Q} U^\dagger$ $+ (\chi^\dagger U - U^\dagger \chi) \mathcal{Q} U^\dagger \mathcal{Q} U \rangle$ | $\frac{1}{8} - Z$ |
| 9 | $\langle d_\mu U^\dagger [\mathcal{C}_R^\mu, \mathcal{Q}] U + d_\mu U [\mathcal{C}_L^\mu, \mathcal{Q}] U^\dagger \rangle$ | $\frac{1}{4}$ |
| 10 | $\langle \mathcal{C}_R^\mu U \mathcal{C}_L^\mu U^\dagger \rangle$ | 0 |

Table 2: Operator basis in the $O(p^2)$ pion-nucleon Lagrangian [38] and in the $O(e^2)$ pion-nucleon Lagrangian [39].

| i | $O_i^{(p^2)}$ | $O_i^{(e^2)}$ |
|-----|---|---------------------------------------|
| 1 | $\langle \chi_+ \rangle$ | $\langle \hat{Q}_+^2 - Q_-^2 \rangle$ |
| 2 | $-\frac{1}{4m^2} \langle u_\mu u_\nu \rangle (D^\mu D^\nu + \text{h.c.})$ | $\langle Q_+ \rangle \hat{Q}_+$ |
| 3 | $\frac{1}{2} \langle u_\mu u^\mu \rangle$ | $\langle \hat{Q}_+^2 + Q_-^2 \rangle$ |
| 4 | $\frac{i}{4} \sigma^{\mu\nu} [u_\mu, u_\nu]$ | |
| 5 | $\hat{\chi}_+$ | |
| 6 | $\frac{1}{8m} \sigma^{\mu\nu} F_{\mu\nu}^+$ | |
| 7 | $\frac{1}{8m} \sigma^{\mu\nu} \langle F_{\mu\nu}^+ \rangle$ | |

Further, the LECs l_i and k_i are ultraviolet-divergent

$$l_i = \gamma_i \lambda + l_i^r(\mu), \quad k_i = \sigma_i \lambda + k_i^r(\mu), \quad (\text{A5})$$

where

$$\lambda = \frac{\mu^{d-4}}{16\pi^2} \left(\frac{1}{d-4} - \frac{1}{2} [\Gamma'(1) + \ln 4\pi + 1] \right). \quad (\text{A6})$$

The components of the external sources are defined as

$$v_\mu = \frac{1}{2} v_\mu^0 + \frac{\tau^n}{2} v_\mu^n, \quad a_\mu = \frac{\tau^n}{2} a_\mu^n, \quad s = s^0 + \tau^n s^n, \quad p = \tau^n p^n. \quad (\text{A7})$$

In tables 1 and 2, we collect the operator basis for meson and meson-nucleon Lagrangians. In addition, in table 1, the divergent parts of the LECs l_i , k_i are listed (see Eq. (A5)).

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